

Online Auction Demand

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Abstract: Online Auction Demand

With \$40BB in annual gross merchandise volume, electronic auctions comprise a substantial and growing sector of the retail economy. Using unique data on Celtic coins, we estimate a structural model of buyer and seller behavior via MCMC with data augmentation. Results indicate that buyer valuations are affected by item, seller, and auction characteristics; buyer costs are impacted by bidding behavior; and seller costs are affected by item characteristics and the number of listings.

The model enables us to compute fee elasticities even though there exists no variation in fees in our data. We find commission elasticities exceed per-item fee elasticities because they target high value sellers and enhance their listing likelihood. By targeting commission reductions to high value sellers auction house revenues can be increased by 3.9%. Computing customer value, we find attrition of the largest seller would decrease fees paid to the auction house by \$97. Given the seller paid \$127 in fees, competitive effects offset only 24% of the fees paid by the seller. In contrast, competition offsets 81% of the buyer attrition effect. In both events, the auction house would overvalue its customers by neglecting competitive effects.

Keywords: Auctions, Structural Models, Two Sided Markets, Empirical IO, Bayesian Statistics.

1 Introduction

Commensurate with the ascendancy of the Internet, e-commerce has witnessed explosive growth. According to a US Census Bureau report in Q1-2006, US e-commerce retail sales increased by 25.4% while retail sales across all channels grew at a more restrained 8.1%.¹ Much of this growth is due to online auctions. For example, eBay alone had 222 million confirmed registered users by the end of Q4-2006. These users generated a gross merchandise volume of \$14.4 billion across 610 million auctions, a growth rate of 23%.² This compares to quarterly sales of roughly \$100 billion in the US Grocery industry.³

Concurrent with this growth, empirical/econometric research pertaining to the design and conduct of auctions has seen increased attention in marketing of late (Chakravarti et al. (2002)). For example, Park and Bradlow (2005) analyze “whether, who, when and how much” to bid under an online auction context; Chan et al. (2007) consider bidders’ willingness to pay for an auction; and Bradlow and Park (2007) investigate how bidders’ behaviors evolve over the course of an auction. This research has led to some important insights regarding the nature of bidder behavior in auctions. We extend this empirical literature by considering the behavior of sellers at the auction site. An integrated analysis of bidder and seller behavior has pivotal implications for the marketing policies of the auction house, such as the fees the auction house sets, the rules used to conduct the auction, or an assessment of the value of its customers (Greenleaf and Sinha (1996)).

Given our interest in seller behavior and its attendant policy implications, we focus on structural models of auction behavior. Such models enable one to ascertain unobserved characteristics of bidders and sellers, such as their latent costs of bidding and listing and their ramifications for auction fees, mechanism design and/or customer value.⁴ Like the marketing literature to which we alluded above, structural model research in the context of auctions has largely focused on bidder behavior (Reiss and Wolak (2005)). For example, Bajari and Hortacsu (2003) propose a structural model to explore the determinants of bidders bidding behaviors. The paper assesses the effects

¹Resource: US Census Bureau(2006, Q1), *Quarterly Retail E-Commerce Sales*. (<http://www.census.gov/mrts/www/ecom.html>)

²Resource: eBay Inc. 4qtr-2006 Financial Releases.

³Resource: “Grocery Stores and Supermarkets Industry Profile Excerpt”, First Research, 2005. (<http://www.firstresearch.com/Industry-Research/Grocery-Stores-and-Supermarkets.html>)

⁴As discussed in Laffont and Vuong (1996), auction data are well-suited for structural models as auctions are asymmetric information games and the data generating process is strategy-driven. For a more complete summary of structural models of auctions, see Reiss and Wolak (2005).

of endogenous bidder entry into an auction and is among the first to model structural bidding behavior in the context of electronic auctions. As such, it forms a cornerstone of the bidder model in our research. Jofre-Bonet and Pesendorfer (2003) consider dynamic bidding behavior arising from capacity constraints in a repeated procurement auction game. They find these constraints lead to higher bids because a winning bid increases capacity constraints in the subsequent period; this reduced incentive to win is commensurate with higher bids. Campo et al. (2002) investigate an auction model with possible risk averse bidders and propose a semiparametric estimation procedure to test the risk neutrality of bidders. There is also a rich literature focusing on the question of identification of empirical auction models, e.g., Paarsch (1992), Guerre et al. (2000), Athey and Haile (2002), Hong and Shum (2002) and Haile et al. (2003). In sum, this literature has further enriched our insights regarding strategic buying behavior in the context of auctions.

Our model supplements the structural auctions literature in a number of regards: First, we integrate both bidder and seller behavior; that is, we do not presume seller behavior regarding the number of items to list to be exogenous. In practice, both bidders and sellers are strategically interactive, and it is reasonable to suspect that the market equilibrium is determined by their interactions. The integration of bidder and seller behavior in auctions is an example of a two-sided market, wherein multiple parties interact on a platform (Rochet and Tirole (2006); Tucker (2005)). Following Rochet and Tirole (2006), the two-sided network as well as the period payment flows between the various agents can be depicted graphically as indicated in Figure 1a. Though there is a rich theoretical literature pertaining to these two-sided markets, empirical research remains nascent (Roson (2005)). We contribute to this empirical literature by a) assessing how these markets should be priced, b) the value of agents in these markets, and c) an empirical analysis of two-sided markets in the context of auctions. Second, we accommodate heterogenous bidding disutilities/costs across bidders.⁵ As a result, we can infer how changes in the listing behavior of the seller affect each bidder’s likelihood of bidding in any given auction. Summing these behaviors across bidders and auctions yields the total auction revenue conditioned on seller listings. Third, we incorporate heterogeneity in seller costs. Heterogeneity in costs implies that the number of

⁵Bajari and Hortacsu (2003) assume an opportunity cost for bidders such that all bidders’ entry decisions satisfy a “zero profit condition”. That is, no bidder obtains positive expected utility for attending auctions. Given the estimates of their model, the distribution of the cost is simulated based on the “zero profit condition” and the model estimates. In our context we observe bidders’ bidding histories across multiple auctions which enables us to explicitly estimate heterogenous bidding disutilities across bidders.

auctions listed can change in response to bidders' valuations or auction house fees. Fourth, we integrate Bayesian statistics and structural models in the context of auctions, which is relatively novel in marketing and economics.⁶ The Bayesian approach enables considerable flexibility in model specification (Rossi et al. (1996)).

Together, these innovations enable us to assess how changes in auction house strategies (such as fee schedules or auction design) affect the number of auctions and the corresponding bidding behaviors even in the absence of any observed variation in these strategies. This in turn affects the number of items upon which bidders bid. As a result, the auction house can forecast the effect of fee changes on the equilibrium number of items listed by sellers and the prices that buyers pay for these items. Closing prices and the number of items listed factor into the auction house revenue. In this manner it is possible to compute price elasticities for the auction house fees in order to evaluate its pricing strategy. In contrast, it is difficult to assess these elasticities by regressing seller listings and closing prices on fees, because there is often little variation in fees from which to infer changes in auction demand and prices. eBay, for example, changes its fees about once per year, leaving few observations from which to infer price response. Moreover, to infer price response, one would need to use observations regarding fee changes that are many years old and it is unclear whether data from the distant past remain valid given the changing sample composition of customers over time.

Using the imputed price-demand system, we conclude (via a pricing policy experiment wherein we manipulate auction house prices) that changing fees can increase auction house revenue by 3.9% with a targeted pricing strategy and 2.9% with a uniform pricing strategy. Much of this gain arises from emphasizing per item fees over commissions. Lower commissions disproportionately attract higher valuation items by increasing per item profits. This further suggests that categories with a greater prevalence of high value items such as art should emphasize per item fees over commissions. Figure 1b depicts the pricing policy experiment graphically in the context of the two-sided market.

A corollary benefit of the foregoing innovations is that they enable one to assess the short-term value of a customer in a two-sided market.⁷ Information regarding customer value is useful for assessing how much to invest in retaining a customer (exemplified via a targeted coupon or rebate from the auction house). A common approach toward assessing customer value is to simply tally

⁶Bajari and Hortacsu (2003) is a notable exception.

⁷By short-term, we refer to the value of the customer over the duration of our data. Long-term, or life-time value would consider the infinite horizon value of a customer.

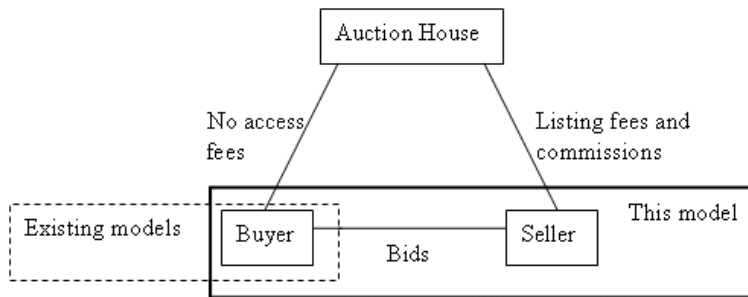


Figure 1a: Payment Flows in a Two-sided Market

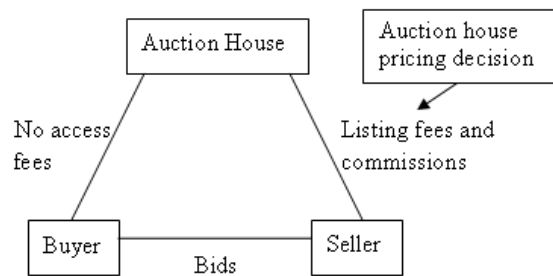


Figure 1b: Pricing Decision of the Auction House in a Two-sided Market

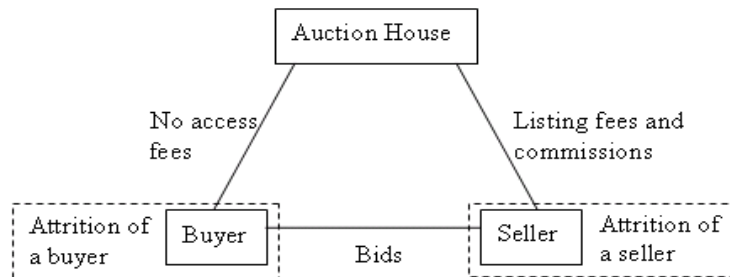


Figure 1c: The Value of a Customer in a Two-Sided Market

the total commissions and fees the seller pays to the auction house. However, were that seller to depart the system, some bidders would switch to other sellers. Moreover, with less competition, the remaining sellers are inclined to list more items. Both behaviors affect equilibrium prices. A proper accounting of these competitive effects offsets about 24% of the fees paid by a departing seller. An analogous argument can be constructed for valuing buyers. The lost revenue resulting from the attrition of a buyer is offset by the remaining bidders who bid on the items that the attriting buyer would have purchased. In addition, the departure of a buyer can incent a seller to reduce listings in response to a decrease in the expected price they will receive. We find other bidders bidding on the items of the attrited buyer offset about 81% of the lost revenue from the attrited buyer. Stated differently, valuing agents without regard to competitive interactions overstates the value of the seller by 1/3 and the value of a buyer in excess of 400%. These conclusions arise from a policy experiment wherein we assess the impact of an attrited customer on equilibrium revenue. This policy experiment is reflected graphically in Figure 1c. It is worth noting that Figures 1a–1c suggest that our model could also be useful in addressing further policy experiments such as assessing the profitability of auction fees to the buyer.

In sum, explicitly considering seller and bidder behavior in a joint system or two-sided market enables one to a) attain a better understanding of how sellers make listing decisions and b) engage policy simulations to help the auction house implement its marketing strategies. With these goals in mind, the paper is organized as follows. Section 2 describes the bidding mechanism used by many online auction houses. This characterization motivates the structure of the game. Next, we present our data and some exploratory analyses to develop insights regarding auction behavior, especially with respect to the interaction between bidders and sellers. We then detail the model and corresponding estimation approach in Section 3 and 4. Results of this model are detailed in Section 5. Using the results, Section 6 overviews the managerial implications, including the impact of fee changes, valuing customers and gauging the effect of seller’s reputation in auctions. This discussion is followed by Section 7, which concludes the paper with a summary of findings, limitations and future research directions.

2 The E-auction Context and Data

A necessary precursor to modeling auction behavior is a complete characterization of the process of listing and bidding an item. Hence, in this section, we preface our model discussion with a characterization of the data and auction mechanism. As the firm supplying the data wishes to remain anonymous, we describe the process in somewhat general terms, beginning with the decisions of the seller.

2.1 Rules of the Auction House and Participants' Decision Processes

2.1.1 Listing

A seller commences an auction by listing an item online. To list an item, the seller must be registered with the auction firm and pay a small listing fee. For an additional fee, sellers can also opt for listing features such as product pictures, the duration of the auction, a secret reserve price, etc.. Secret reserves enable a seller to retain the listed item should the highest bid fail to exceed the reserve, which is not revealed to bidders. In addition to these listing features, experienced sellers (by virtue of interactions with past buyers) can also garner reputation ratings. Previously successful buyers can provide positive, negative or neutral ratings for the seller based on the buyer's experience with the transaction.

If the auction successfully concludes, the seller pays a commission and listing fee to the firm. If not, the seller is responsible only for a listing fee. Other seller costs include acquisition costs and shipping. We presume sellers' listing strategies (whether and how many items to list) are selected to maximize the seller's profit. Figure 2 overviews the decision of sellers. The links in the Figure, denoted S1-S8 reflect the processes modeled in this paper.

2.1.2 Bidding

Bidders typically initiate the bidding process via a key-word search to locate relevant sellers, categories and items of interest. The results of this search list pertinent items open to bidding, sorted by price and time left until the auction concludes. Upon finding an item of interest, a bidder can bid immediately (if registered) or place the item onto a "watch list" for subsequent monitoring and potential bidding.

The bidding mechanism used by the auction house is called "proxy bidding". With proxy

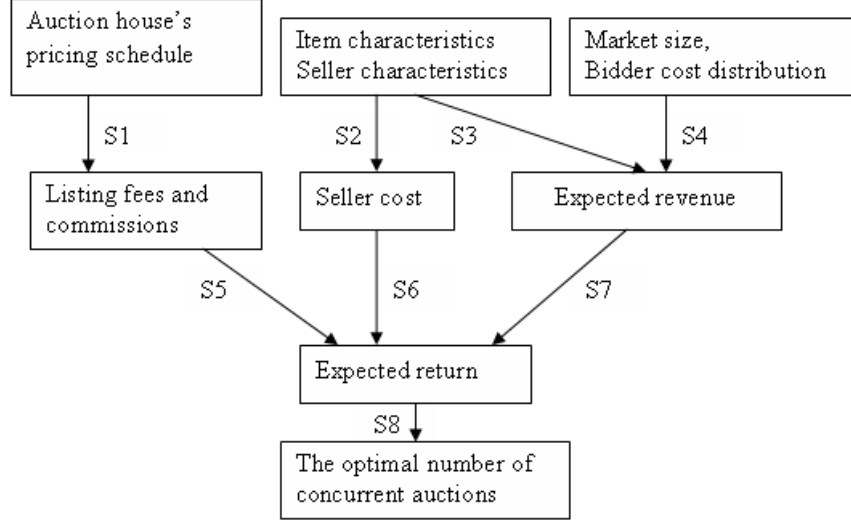


Figure 2: The Seller Decision

bidding, a bidder enters an auction prior to its conclusion and submits their bid. The website will then act as a “proxy” to bid for the bidder by entering a bid on behalf of the user whenever the user is outbid (to some pre-determined maximum level). For example, assume a bidder intends to bid no more than \$10.00 for a given item. Suppose further that the item’s current price is \$1.00 with a bid increment of \$0.50. By submitting a \$10.00 proxy bid to the website, the auction site enters a bid of \$1.50 on behalf of the bidder. If another bidder enters and bids \$5.00, the proxy bid automatically becomes \$5.50. If a subsequent bid of \$15.00 materializes, the high bid changes to \$10.50 and the first bidder receives an E-mail notification that they were outbid. The bidder can then choose whether to increase their bid or quit the auction altogether.

Upon placing the highest bid, the bidder wins the auction and makes a payment to the seller that equals to the second highest bid. It is the responsibility of the seller and the winning bidder to settle payment and delivery issues. We presume that a bidder’s bidding strategy (whether and how much to bid) is selected to maximize their expected profits. More specifically, we assume the bidder will bid if the expected return from doing so exceeds the cost. Figure 3 overviews bidding decision process. The links in the Figure, denoted B1-B10 reflect the bidder behaviors we model.

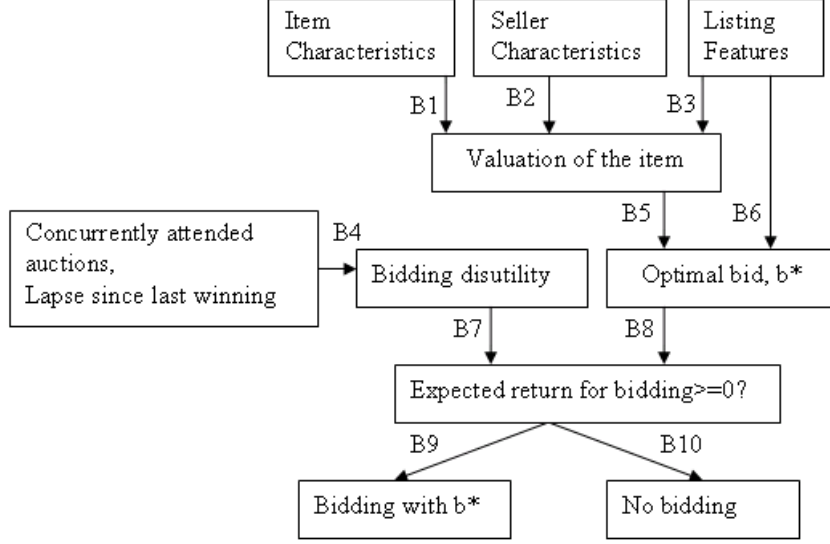


Figure 3: The Bidder Decision

2.1.3 Seller-Bidder Interactions

The foregoing discussion suggests why it is desirable to model bidder and seller behaviors jointly. First, the bidder’s costs and returns affect the optimal bidding levels and decisions to bid. Strategic sellers form expectations regarding bidding levels and bidder costs. Predicated upon these expectations, sellers make listing decisions. Bidders, who are also strategic, in turn make bidding decisions conditional on the seller’s listing behavior. Hence, seller and bidder decisions are interdependent. Accordingly, the auction house directly affects the seller’s profits and the number of items sellers list by varying auction fees. The effects of a fee change propagate to bidders by altering the decisions of sellers. To exemplify this point, we next describe our data and present some descriptive statistics regarding bidder-seller interactions.

2.2 Data

We use a unique data set generously presented by an international auction house that prefers to remain anonymous. The focal category is collectible Celtic coins. We selected this category because it is fairly isolated, inasmuch as bidders in this category tend not to buy other types of coins. This mitigates considerations regarding seller and bidder behavior in other categories. The data were

collected from November, 2004 to April, 2005 and encompass 816 auctions⁸ listed by 57 sellers over 189 days. Of these listings, 72.2% were finally sold. The number of bidders is 925. The auctions data comprise several files, including a listing file, a bidding file, and a demographics file for the bidders and sellers. We describe each in turn.

2.2.1 Listing File

The listing data include, for each item listed, the unique seller ID, a text description of the item, and the item’s listing characteristics. These characteristics include “picture”, whether the seller includes at least one picture of the item in their listing; “subtitle”, a paid feature which present detailed descriptions under the listing title; “gallery picture”, an icon-size picture of the item beside the title when the auction is presented by the search engine; “store”, which enables a seller to group items on a single web page; “bold”, in which the auction title is shown as bold characters; and “featured listing”, wherein the item listing is displayed near the top of the searching results. For each listing, we also observe the exact value of the secret reserve price, if chosen by the seller. We include the percentage statistics of the listing features in Table 1. This Table shows “Bold”, “Featured Listing” and “Secret Reserve” are seldom used by sellers.

Table 1: Listing Features Summary Statistics

Listing Features	Percentage
Picture	90.21
Subtitle	4.57
Gallery Picture	30.90
Bold	1.85
Featured Listing	0.54
Online Store	28.00
Secrete Reserve	2.33
Number of Auctions	816

To obtain the prevailing market prices of listed items, we collect the data from two sources. The first one is an online coins catalog (www.vcoins.com), which is widely recognized among the coin collectors community. A second source is from the book *Coins of England and the United Kingdom*,

⁸This number reflects the exclusion of 36 largely inactive sellers who only listed one auction in the 6-month period. The sparse observation per seller make it difficult to make inferences regarding their behavior and identify seller-specific effects (that said, our key results are robust to their inclusion). By omitting these 36 sellers/auctions, we also lose 10 bidders (out of a pool of 935 bidders) who bid only in those auctions.

Standard Catalogue of British Coins, 41st Edition. We report the price information together with some other listing information in Table 2.

Table 2: Auctions Summary Statistics

	Mean	Std. Dev.	Median	Min.	Max.
Book Value \$	93.39	96.87	55	3	675
Final Price/Book Value	0.86	1.31	0.48	0	4.25
Secret Reserve Price/Book Value (Obs.=19)	1.33	2.23	0.54	0.13	9.06
Minimum Bid/Book Value	0.15	0.46	0.1	4.00E-05	11.65
Average Listing Fees per Auction \$	0.74	1.59	0.35	0.25	21.40
Average Commissions per Auctions \$	1.00	1.99	0.39	0	22.62
Duration (Days)	7.01	1.14	7	1	10
Lapse Since Last Listing (Days)	53.23	42.22	42	1	180
Number of Concurrent Listings By the Same Seller	1.65	1.3	1	1	12
Number of Sellers per Week	14.96	3.84	15	9	23

The final price is close to book value. While auction durations vary from 1 day to 10 days, most auctions last 7 days. Hence, we define the interval of analysis (e.g., whether to list) to be weekly. All auctions employ an open reserve price, denoted minimum bid, that is observed by all buyers. A handful of sellers (2.3%) also employ a secret reserve. Though its level is not known to the bidders, the presence of a secret reserve is common knowledge and can therefore enter the bidder utility function.

2.2.2 Bidding File

The bidding data include a detailed bidding history of each unique bidder ID through the 6 months. Thus we observe every bid a bidder submits and the time the bid was made. This includes the highest bid that occurred in each auction, which is not normally observed for most sealed second-price online auction data. Table 3 presents some summary statistics of the data.

Table 3: Bidders Summary Statistics

	Mean	Std. Dev.	Median	Min.	Max.
Number of Bidders per Auction	2.58	2.94	1.50	0	18
Number of Bids per Auction	4.17	5.44	2	0	31
Lapse Since Last Winning (Days)	61.37	41.73	56	7	189
Number of Concurrent Auctions Attended per Week	1.02	0.24	1	1	15
Number of Bidders per Week	64.46	29.82	58.50	18	160

The majority of bidders only attend one auction per week, suggesting that purchases are not concentrated in the hands of a few buyers. We also observe little evidence of buyer reselling as none are cross listed as sellers and they tend to purchase only one coin at a time. In addition, the total number of items listed is dominated by a few large sellers.⁹ Together, these observations reflect a market comprised of many collectors buying from a set of dealers and that it is therefore appropriate to model the bidders and sellers as different agents. The lapse since last winning varies dramatically across bidders and suggests the importance of capturing heterogeneity across bidders.

2.2.3 Demographic File: Seller and Buyer Feedback

The demographic file includes demographic information for sellers and bidders. With the exception of participants’ feedback scores, these demographics are incomplete, so we focus only on feedback scores. We report the feedback scores in Table 4. Among the participants population, 5.5% are females, 53.6% are males, and the balance did not report their gender.

Table 4: Feedback Summary Statistics

	Mean	Std. Dev.	Median	Min.	Max.
Seller Feedback Rating	1886.25	2554.18	821.5	1	13600
Bidder Feedback Rating	186.55	273.55	89	-1	2167

While feedback scores for both sellers and bidders evidence considerable variation, the scores for sellers are more diverse. Also, as a group, sellers have a higher mean and median feedback score than bidders, which suggests that sellers are more active and experienced than bidders.

3 An Integrated Model of Bidders and Sellers

3.1 Key Assumptions and Nomenclature

We detail a number of assumptions used to make our modeling approach more efficient. Most are standard assumptions in the literature. These are as follows:

- First, we assume a private value (PV) auction. The assumptions of a private value (PV) auction and a common value (CV) auction lead to different interpretations of the data and

⁹The seller market is moderately concentrated with a Herfindahl Index of 0.12. The top 15 sellers account for 80% of all listings.

methods of inference (Milgrom and Weber (1982)).¹⁰

Table 5: Private Value vs. Common Value —Regression of Bids on the Number of Bidders

	All Bids	Last Bids
Intercept	-0.05 (0.16)	0.34 (0.21)
Log(Number of Bidders)	0.48* (0.03)	0.41* (0.04)
Log(Book Value)	0.65* (0.03)	0.57* (0.04)
Log(Seller Feedback)	-0.06* (0.01)	-0.05* (0.02)
Secret Reserve Price	0.49* (0.10)	0.56* (0.15)
Gallery Picture	0.03 (0.04)	0.10* (0.05)
Log(Bidder Feedback)	0.01 (0.01)	0.01 (0.02)
Number of Obs. Used	3344	2117
Adjusted R^2	0.29	0.25

To justify the PV assumption, we use the empirical test first proposed by Milgrom and Weber (1982). The method is also discussed and implemented in other literature (Athey and Haile (2002); Bajari and Hortacsu (2003); Haile et al. (2003); Paarsch (1992)). The idea of the test is to exploit the relationship between the number of bidders and bids. With the existence of the “winner’s curse” in common value auctions, the average bids in an N -bidder auction should be lower than a $(N - 1)$ -bidder auction.¹¹ In comparison, such a relationship does not hold under a private value assumption. In Table 5, we present the results from two IV regressions of (log) bids on (log) number of bidders. The first regression uses all bids while the second only uses the last bids of each bidder. Using (log) minimum bid, “Bold” and “Featured Listing” as instruments for the number of bidders, the regression coefficients for the number of bidders on prices are positive and significant. Thus we proceed with the Private Value assumption in our structural empirical model. Under the PV assumption auctions reduce to a second price, sealed bid auction (Vickrey (1961); Milgrom and Weber (1982)).

¹⁰There are two types auctions based on the degree of independence across bidders’ valuation for an auctioned item: private value and common value. In a private value model, bidders evaluate the item independently from others and such valuations are private information. Knowledge of others’ valuations does not affect one’s own valuation. In contrast, the common value model assumes all bidders’ valuations are identical ex post (even though each bidder may have an idiosyncratic ex ante valuation).

¹¹In common value settings, winning bidders are those most likely to have over-estimated an item’s value (which is the reason for their highest bids). Note that all bidders have the same ex post valuation. Thus, winning bidders typically over-pay, hence the “winner’s curse”. The bidder will lower their bid to offset this potential over-estimation.

- Second, we assume bidders' private valuation signals for a given item are drawn independently from the value distribution, another common assumption in the literature, denoted independent value (IV). Given the context of bidders bidding modestly priced items over the Internet (with dispersed bidders and limited interpersonal contact), we also believe this to be a reasonable assumption. Note that this assumption does not imply valuations are independent as changes in the mean of the distribution affect all bidders. For example, the seller use of a gallery can affect all buyers' valuations. As noted by Reiss and Wolak (2005), the alternative assumption of Affiliated Values (AV) has seen scant attention because it is difficult to characterize the equilibria of these auctions (requiring additional strict assumptions). Thus, like most research that precedes ours, we model an Independent Private Value (IPV) auction on the bidder side.
- Third, we consider a static game with bidders and sellers. This assumption is not as restrictive as it initially appears, as we can control for dynamics such as an inter-temporal budget constraint via a reduced-form approximation. We leave more formal resolution of the dynamic problem for future research; this would entail solving a dynamic program, ensuring the equilibrium is unique, and identifying this solution from the data, all of which may be a contribution in its own right, if feasible. As noted in our literature review, this is a prevalent assumption.
- Fourth, we model cross-auction interdependence by specifying a diminishing marginal value for each additional auction attended. In contexts wherein bidders routinely bid on multiple concurrent auctions, there exist the potential for other interdependencies in strategic bidding behavior (Brusco and Lopomo (2006)). However, Table 3 indicates there exists little multiple auction bidding suggesting that this is an appropriate approximation. Accommodating strategic cross-item bidding adds considerable complexity to the model with little attendant benefit given the limited occurrence of these events in our data.
- Fifth, we assume the sellers' choices of listing features (e.g., reservation prices, gallery, etc.) are exogenously given. Endogenizing this decision admits a multiplicity of listing feature equilibria, rendering such a specification of little value for assessing how the pricing of features affects demand for these features. Moreover, endogenizing feature choices implies the need to

compute an equilibrium for every set of features, which leads to the curse of dimensionality and quickly becomes infeasible (or requiring an approximate as opposed to an exact solution). Thus, we leave it to future research in combinatorial optimization to assess whether this problem is resolvable and restrict our analysis to the effect of listing fees and commissions.¹² We note prior research considers seller behavior to be altogether exogenous.

- Sixth, our model estimation assumes bidders and sellers are fully informed about the number of bidders in the market and the bidder valuation and bidder cost distributions. The assumption that bidders and sellers are aware of the value distribution may be a reasonable approximation in light of the ability of bidders and sellers to monitor auction outcomes over time by observing historical bids on the web site. Most preceding research makes this assumption on the bidder side and we extend this precedent to the seller side. The number of sellers and bidders can also be observed from the web site. We explore the assumption that sellers and bidders are fully informed about the buyer cost distribution by estimating a model wherein we assume their knowledge is limited only to the mean of this distribution. We find this model leads to a 0.9% decrease in the log marginal likelihood.¹³ For our policy simulations we invoke the additional assumption sellers form expectations about the number of sellers in the market and the distribution of seller costs.
- Seventh, we assume risk neutral and symmetric bidders, consistent with previous literature (e.g., Bajari and Hortacsu (2003)). Symmetry implies that bidders draw private valuations from the same distribution, *ex ante*. However, upon receiving their signals, they differ in their individual valuations. Moreover, heterogeneity in bidder costs implies bidders are asymmetric in their bidding utilities even when they *ex ante* have the same expected valuation for an item (as utility is value less costs). Accordingly, the symmetry assumption is not as restrictive as

¹²It is possible to endogenize the reservation price decision. A benefit of this approach is that such a model formalizes the decision making process on the part of the seller. The cost is that endogenous reservation prices make the model more complex (i.e., introduces additional noise along with explanation). Given the trade-off, we test such a model and find that endogenizing reservation price has only a negligible effect on parameter estimates (the correlation between the median parameter estimates in this model and our base formulation is 0.985). Further, the endogenous reservation price model evidences lower fit (by decreasing the log marginal likelihood for listings and bids from -21278.41 to -22150.82). Therefore we do not pursue this development further in this paper. As an aside, endogenizing reservation prices leads to reservation pricing equilibria that may not be unique; as such the model has more limited applicability in the context of policy simulations.

¹³The assumption that bidders and sellers know the full bidder cost distribution results in a log marginal likelihood of -21278.4. Restricting this knowledge to only the the mean of this distribution yields a log marginal likelihood of -21469.9. Hence the data support the assumption that bidders and sellers are aware of the bidder cost distribution.

it may appear. Nonetheless, we explore this assumption following the approach of Athey et al. (2004) and Flambard and Perrigne (2004), who posit asymmetry in valuations arises from bidders' idiosyncratic characteristics. Following this approach, we regress final bids on observed bidder characteristics including bidder feedback, time lapse since last bid, and time lapse since last win. We find none of these effects to be statistically significant, consistent with the assumption of symmetric bidders.

- Eighth, we assume that the focal auction house is operating as a monopoly. We believe it is a reasonable approximation due to the dominant market share of the auction house in the category and market we consider. Its scale leads to strong network effects (for example, a seller can reach a large number of bidders) making it difficult for bidders and sellers to successfully buy and list items with competing houses and reducing the likelihood they will defect over a small change in fees. Evidence for this assumption is afforded by a small share competitor who lowered their fees with no effect on the share of the considered firm.
- Ninth, we focus on behaviors within one category and abstract any cross category bidding, listing implications (e.g., Haile et al. (2003) and others). We select our category of analysis to comport with this assumption.

3.2 Model Overview

We assume the bidder-seller game in any given period (e.g. weekly in our data) as follows:

- In Stage 0, each seller is endowed with some types of coins associated with a seller-item specific opportunity cost for acquiring the coin(s). For each type of these coins, an optimal listing feature combination is *exogenously* determined by the item's and seller's characteristics. This combination of listing features, in conjunction with seller and item characteristics, determines the distribution of valuations of bidders regarding the item. Each bidder will then draw a valuation for the item from this distribution.
- In Stage 1, conditioned upon house fees, the acquisition costs and the expected revenue, the seller assesses the expected return. The seller then decides whether to list and, if so, how many items to list for each type conditional on the expected returns. More specifically, we

presume the seller lists the precise number of items that leads to the highest profit over all alternative numbers of items listed.

- In Stage 2, for each auction, each bidder decides whether to participate in the auction and, if so, submits the optimal bid. As noted by Bajari and Hortacsu (2003), sniping (last-minute bidding) is the unique Bayesian Nash equilibrium for on-line auctions and, as a result, these auctions are equivalent to sealed-bid second price auctions. Even in the absence of sniping, when the IPV assumption sustains, auctions are equivalent to sealed-bid second price auctions (Vickrey (1961); Milgrom and Weber (1982)), i.e., the timing of bid submission does not affect the optimal bidding strategy. As such, we abstract away from within-auction bidding dynamics.

Given we assume that the listing feature choice and cost function are exogenously determined in Stage 0, we only detail the game for Stage 1 (how many items to list) and Stage 2 (conditioned on listings, how much to bid). We solve this problem using backward induction, beginning with the bidder model first. As a further prelude to our model, some clarifications of the indices we use are necessary. We define $i = 1, 2, \dots, I$ to indicate sellers and $j = 1, 2, \dots, J$ to denote the bidders in the market. Different types $t = 1, 2, \dots, T$ of items are available for auction (e.g., Celtic silver drachm of Alexander III vs. Celtic silver tetradrachm of Philip II). The same seller can initiate multiple auctions for the same type of coin. We use $n = 1, 2, \dots, N$ to index auctions.

3.3 The Second Stage – Bidder’s Model

3.3.1 The Valuation of A Bidder

For the n -th auction with item t listed by seller i , each potential bidder draws a valuation from the distribution with p.d.f. $g(v_{ijtn}|\mu_{it}, \sigma)$, where v_{ijtn} stands for bidder j ’s valuation for the item (for example, a specific coin). We presume the bidder’s valuation for a given type of item auctioned by a given seller remains the same across auctions, i.e., $v_{ijtn} = v_{ijtn'}$. This ensures that the primary source of variation in unobserved valuations occurs across bidders as opposed to within bidder volatility in valuations, as the within person variation is likely to be relatively constant over a short span of time (recall, we can allow longitudinal variation in bidder valuations via observed changes in item and seller characteristics in conjunction with listing features). We define μ_{it} and σ to be the value distribution mean and standard deviation respectively. A natural candidate for

the distribution $g(v_{ijtn}|\mu_{it}, \sigma)$ is a normal distribution. We assume that¹⁴

$$(v_{ijtn}|\mu_{it}, \sigma) \sim N(\mu_{it}, \sigma) \quad (1)$$

For second price sealed auctions under an independent private value setting, one bidder’s valuation of the item is independent from others’ valuations and the number of competitors. Upon winning the auction, the realized gross return is exactly the bidder’s valuation, v_{ijtn} .

We posit the valuation distribution of an auction is incumbent upon item and seller characteristics as well as listing features (links B1-B3, Figure 3). We therefore set

$$\mu_{it} = Z_{it}^{\mu} \beta^{\mu} \quad (2)$$

$$Z_{it}^{\mu} = [1, BOOKVAL_{it}, FeedbackSeller_i, MinBid_{it}, ReserveDummy_{it}, StoreDummy_i, SubtitleDummy_{it}, GalleryDummy_{it}] \quad (3)$$

and ensure σ to be positive by using a truncated normal as its proposal density function in the sampling chain (see the Appendix A3.3).¹⁵ The choice of variables to use is motivated by the variables available in the data. However, there is little variation in “Bold” and “Featured Listing” so it is not possible to reliably estimate these effects.

We place normal distributions on the priors of β^{μ} . We try to minimize the effect of the choice of priors on the shape of posterior distributions by choosing a diffuse prior. Details of the choice of priors and sampling chain are presented in the Appendix A3.

3.3.2 Minimum Bid, Disutility of Bidding and The Optimal Bidding Strategy

Optimal Bidding Strategy In addition to bidder valuations (link B5), two other factors affect bidder strategy. First, each auction has a minimum bid, $MinBid_{it}$, which is common across the same items listed by a seller. $MinBid_{it}$ functions as a “reserve price” as discussed in Milgrom and Weber (1982). All submitted bids must be greater than the minimum bid (link B6). Second, each customer has a disutility of bidding, C_{ijtn}^b , which we express as a dollar metric and can be

¹⁴We also considered a lognormal distribution for v to ensure all valuations are positive, however this led to a considerable decrease in model fit (-21278.4 vs. -23305.8). The negative values implied by the normal distribution may be justified by the explanation that some bidders do value the item negatively. A similar specification is used in Bajari and Hortacsu (2003).

¹⁵We also considered a specification wherein the variance of the bidder valuation distribution was a function of item, seller and marketing characteristics, that is $\sigma_{it} = \sigma(Z_{it})$. However, this model did not improve fit nor were any of the variables significant (the log marginal likelihood of the alternative specification is -21355.7).

interpreted as the bidder’s “marginal cost” for bidding on one more item (we parameterize these costs in the next section). Higher costs reduce the likelihood of a bid on a given auction (link B7). This disutility can involve opportunity costs on time such as the efforts for researching and monitoring of the auction as well as reflecting the opportunity cost of capital. Following an approach similar to Milgrom and Weber (1982), we obtain the following result.

Theorem 1 *For a given auction itn having a minimum bid $MinBid_{it}$, bidder j has bidding disutility C_{ijtn}^b . The optimal bidding strategy is*

$$b^*(v_{ijtn}) = \begin{cases} v_{ijtn} & \text{if } v_{ijtn} \geq x_{ijtn}^* \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where x_{ijtn}^* is defined by the implicit function

$$C_{ijtn}^b = \int_{-\infty}^{x_{ijtn}^*} (x_{ijtn}^* - \max(\alpha_{\bar{j}}, MinBid_{it})) f(\alpha_{\bar{j}}) d\alpha_{\bar{j}} \quad (5)$$

where $f(\alpha_{\bar{j}})$ is the density function of $\alpha_{\bar{j}}$, the highest competing bid.

Proof: See Appendix A1.

Theorem 1 suggests that in equilibrium a bidder will only bid if their valuation for an item exceeds a certain threshold, x_{ijtn}^* , and, if so, they will bid the valuation, i.e., “truth-telling” (links B8-B10). The bidding threshold, x_{ijtn}^* , is an increasing function of three factors: i) the $MinBid_{it}$, ii) the bidder cost, C_{ijtn}^b and, iii) the expected highest competing bid, $\alpha_{\bar{j}}$. As the $MinBid_{it}$ increases, only higher value bidders will participate, increasing the expected closing price. Hence only higher valuation bidders are likely to participate thereby elevating the participation threshold. Second, higher bidder costs imply higher valuations are necessary to make a bid profitable. Thus, the bidding threshold increases with bidder costs. Third, x_{ijtn}^* is increasing in $\alpha_{\bar{j}}$. As an order statistic, $\alpha_{\bar{j}}$ increases with the number of bidders; hence the bidding threshold rises with the number of bidders. As such, the intensity of competition affects the likelihood of an individual’s entry into an auction.

Bidder Costs, C_{ijtn}^b The marginal disutility of bidding in a focal auction (link B4) is assumed to be bidder-auction specific and is specified as follows:

$$\begin{aligned} \log(C_{ijtn}^b) &= \beta_{0j}^{C_b} + Z_{ijtn}^{C_b} \beta^{C_b} + e_{ijtn}^{C_b}; \quad e_{ijtn}^{C_b} \sim N(0, 1) \\ Z_{ijtn}^{C_b} &= [\log(AttendedAuction_{ijtn} + 1), \log(Lapse_{ijtn})] \end{aligned} \quad (6)$$

where *AttendedAuction* is the total number of alternative auctions the bidder is attending during the duration of the focal auction; the “+1” ensures the log function is defined; *Lapse* is the lapse of time since the last win; and $var(e_{ijtn}^{C_b}) = 1$ to ensure identification of costs.¹⁶

The *Lapse* variable reflects potential inter-temporal dynamics in bidding behavior. Inter-temporal dependence can also be induced by price or listing expectations. For example, rising expectations regarding the number of potential future listings might induce customers to delay purchases. In addition, the presence of an inter-temporal budget constraint or an inventory constraint implies bidders who recently win auctions may have less space or cash available to purchase on subsequent bidding occasions, thereby lowering their bidding likelihood (i.e., the marginal utility of bidding decreases as the lapse in time since a win decreases). Similarly, the *AttendedAuction* variable reflects the potential existence of a within-period budget or inventory constraint. In the event bidders have insufficient space or capital to accommodate multiple auction wins, their marginal likelihood of bidding in additional auctions will decrease (else bidders will either bid on all available auctions (if their marginal value of bidding exceeds the marginal cost) or none). $\beta_{0j}^{C_b}$ is a bidder-specific constant to account for heterogeneity in bidder costs. We use log costs to ensure costs are positive. We assume $\beta_{0j}^{C_b}$ has the following hierarchical structure to capture heterogeneity in costs across bidders.

$$\begin{aligned}\beta_{0j}^{C_b} &\sim N\left(\bar{\beta}_0^{C_b}, (\phi^{C_b})^{-1}\right) \\ \bar{\beta}_0^{C_b} &\sim N(\overline{\beta}_0^{C_b}, \sigma_{0C_b}^2) \\ \phi^{C_b} &\sim \text{Gamma}(a_0^{C_b}, b_0^{C_b})\end{aligned}\tag{7}$$

On the surface, it may appear that bidder valuations and costs are not separately identified, as a concurrent increase in both would yield the same bidder profits and thus bidding behavior.

¹⁶To see this, consider equation (A-47) in the Appendix. When variances are not constrained to 1, (A-47) can be rewritten as

$$L \propto \prod_{ijtn} \exp\left(-\frac{(\log C_{ijtn}^b - \beta_{0j}^{C_b} - Z_{ijtn}^{C_b} \beta^{C_b})^2}{2\sigma^2}\right).$$

Because C_{ijtn}^b is not observed, this expression can be rewritten as

$$L \propto \prod_{ijtn} \exp\left(-\frac{\left((\log C_{ijtn}^b)/\sigma - (\beta_{0j}^{C_b}/\sigma) - Z_{ijtn}^{C_b} (\beta^{C_b}/\sigma)\right)^2}{2}\right)$$

from which it can be observed the parameters are identified only up to a scale, analogous to a probit or logit model.

Observations of positive bids reveal the bidder’s valuation, which in turn helps to determine their cost. With costs known, together with our parametric specification regarding the costs, it becomes possible to infer values in auctions wherein bids are not observed (that is, a bidder decide not to bid). In addition, costs have a common parametric specification across different items. For those observations with the same covariate values for bidder costs, the variation in bidder behavior across these items helps to identify bidder valuations because differences in costs cannot explain differences in bidding behavior (as these costs are constant across bidders in this case).

3.4 The First Stage – Seller’s Model

The seller chooses the optimal number of items to list in order to maximize their expected return. We first derive the expected revenue a seller obtains from listing an item (links S3 and S4 in Figure 2), and then use this revenue in conjunction with seller costs (links S1 and S2) to derive the expected return for listing a specific number of items (links S5-S7) and the optimal number of items to list (link S8).

3.4.1 Expected Revenue for Listing An Item

Seller Expected Revenue $E(R_{itn}^s|\cdot)$ Without Secret Reserve Price For auctions without secret reserve prices, the seller receives the second highest bid when there are at least two bidders competing. When there is only one bidder, the seller receives the minimum bid $MinBid_{it}$. If there is no bidder, $E(R_{itn}^s|\cdot) = 0$. We condition the seller revenue on each of these events, as discussed next.

■ **Number of Bidders ≥ 2 .** For the case where there are at least two bidders, denote α as the second highest bid among bidders and x_α^* as the threshold (defined in Theorem 1) associated with the bidder bidding α .

Theorem 2 *Given μ_{it}, σ and market size J , the conditional distribution of α , the second highest*

bid, can be expressed as

$$\begin{aligned}
p(\alpha|\mu_{it}, \sigma, J) &= J(J-1) \cdot \left\{ \int_{C_{ijtn}^b} \{ [G(\alpha|\mu_{it}, \sigma) - G(x_{ijtn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha) \right. \\
&\quad \left. + G(x_{ijtn}^*|\mu_{it}, \sigma) \} f(C_{ijtn}^b) dC_{ijtn}^b \right\}^{J-2} \\
&\quad \times \left\{ 1 - \int_{C_{ijtn}^b} \{ [G(\alpha|\mu_{it}, \sigma) - G(x_{ijtn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha) \right. \\
&\quad \left. + G(x_{ijtn}^*|\mu_{it}, \sigma) \} f(C_{ijtn}^b) dC_{ijtn}^b \right\} \\
&\quad \int_{C_{ijtn}^b} g(\alpha|\mu_{it}, \sigma, \alpha \geq x_{ijtn}^*) f(C_{ijtn}^b) dC_{ijtn}^b
\end{aligned} \tag{8}$$

where C_{ijtn}^b is the bidding disutility associated with the focal auction and $f(C_{ijtn}^b)$ is the distribution density of C_{ijtn}^b ; $F_{x^*}(\cdot)$ is the distribution of x_{ijtn}^* and $F_{x^*}(\alpha)$ indicates the probability of $\alpha \geq x_{ijtn}^*$; x^* is defined in equation 5.

Proof: See Appendix A2.

The term in the first set of braces represents the probability $J - 2$ bids lie below α ; the term in the second row represents the probability that the highest bid is greater than α ; and the term in the third row is the probability the second highest bid is α . The integrals yield expectations over the unobserved bidder costs. The term J reflects the number of permutations in which α can be the second highest bidder. Similarly, the term $(J - 1)$ reflects the fact that any bidder among the $(J - 1)$ bidders (J bidders less the second highest one) can be the highest bidder.

■ **Number of Bidders = 1.** When there is only one bidder, just that bidder's valuation exceeds their threshold while the other $J - 1$ bidders do not. That yields the probability of the seller earning $MinBid_{it}$ as the following

$$\begin{aligned}
\Pr(R_{itn}^s = MinBid_{it}) &= J \cdot \left\{ \int_{C_{ijtn}^b} G(x_{ijtn}^*|\mu_{it}, \sigma) f(C_{ijtn}^b) dC_{ijtn}^b \right\}^{J-1} \\
&\quad \times \left[1 - \int_{C_{ijtn}^b} G(x_{ijtn}^*|\mu_{it}, \sigma) f(C_{ijtn}^b) dC_{ijtn}^b \right]
\end{aligned} \tag{9}$$

That is, for $(J - 1)$ bidders, each has a valuation lower than the threshold, yielding $G(x_{ijtn}^*|\mu_{it}, \sigma)$. This is the term in the first set of brackets on the right hand side. Given that C_{ijtn}^b is a random variable from the seller's perspective, this term must be integrated out. The second term pertains to the one bidder whose valuation exceeds their threshold, leading to $[1 - \int_{C_{ijtn}^b} G(x_{ijtn}^*|\mu_{it}, \sigma) f(C_{ijtn}^b) dC_{ijtn}^b]$. Moreover, as this sole bidder is chosen from among J bidders, the final result needs to be multiplied by J potential bidders that can have the highest bid (that is, $\binom{J}{1}$).

Combining equation 8 and 9 yields the conditional expected return of the seller,

$$\begin{aligned} E(R_{itn}^s | \mu_{it}, \sigma, J) &= E(\alpha | \mu_{it}, \sigma, J) + MinBid_{it} \cdot \Pr(R_{itn}^s = MinBid_{it}) \\ &= \int_{MinBid_{it}}^{\infty} \alpha p(\alpha | \mu_{it}, \sigma, J) d\alpha + MinBid_{it} \cdot \Pr(R_{itn}^s = MinBid_{it}). \end{aligned} \quad (10)$$

Seller Expected Return With Secret Reserve Price The results developed in the case of no secret reserve can be generalized as follows to the case of a secret reserve. First, in the case of multiple bidders, if the second highest bid is lower than the secret reserve, the seller keeps the item. Thus the return is zero. Second, in the one bidder case, the seller retains the item because the winning bid must be $MinBid_{it}$ and is therefore lower than the secret reserve.¹⁷ Hence, the expected return becomes

$$\begin{aligned} E(R_{itn}^s | \mu_{it}, \sigma, J) &= E(\alpha | \mu_{it}, \sigma, J, \alpha \geq Reserve_{it}) \\ &= \int_{Reserve_{it}}^{\infty} \alpha p(\alpha | \mu_{it}, \sigma, J) d\alpha \end{aligned} \quad (11)$$

3.4.2 The Seller Listing Decision

Seller Profits Prior to bidding, the seller must decide whether to list an item, and if so, how many of the items to list.¹⁸ This decision is analogous to the seller choosing the optimal supply of goods at a subgame level. We assume that seller i 's conditional expected profit given listing q_{it} units of item t is:¹⁹

$$\pi_{it}(q_{it}) = q_{it} \cdot (1 - commission) E(R_{itn}^s | \cdot) - C_{it}^s(q_{it}) - q_{it} \cdot fee_{it} \quad (12)$$

where *commission* is the commission rate charged by the auction house; *fee_{it}* is the unit listing fee paid to the auction site.²⁰ Other than the listing fees and commissions paid to the auction house, there exist acquisition or opportunity costs for an item. The cost is assumed to depend on the item's prevailing market value and the number of units listed, q_{it} . Owing to finite supply, as the seller endeavors to source more units acquisition becomes more difficult. This leads to an increase

¹⁷Although the seller can still choose to sell the item if the realized final price is lower than the secret reserve, we assume the seller assigns zero probability to such an event when calculating *ex ante* expected return.

¹⁸We also estimated a model wherein the minimum price decision is endogenous. The endogenous minimum price model yields a lower log marginal likelihood for the listing decision and bids (-21278.41 vs. -22150.82).

¹⁹Under the assumption that listing features are exogenous, a unique equilibrium is assured with a quasi-concave revenue function and a convex cost function.

²⁰Given that commissions are tiered in our data, we compute commissions incumbent upon the auction's closing price. However, to facilitate exposition, we omit tier-specific commission subscripts throughout the paper.

in the marginal opportunity cost of obtaining these items. Denote $C_{it}^s(q_{it})$ as the total acquisition cost for listing q_{it} units. Increasing marginal costs imply that $C_{it}^s(q_{it})$ is convex in q_{it} . Moreover, our specification implies that the expected revenue is quasi-concave in q_{it} , which is a necessary condition for a concave profit function and ensures an optimum exists. The seller's problem is to find the optimal q_{it} that satisfies

$$q_{it} = \arg \max_{\tilde{q}_{it}} \pi_{it}(\tilde{q}_{it}) \quad (13)$$

i.e., any deviation from q_{it} (i.e., selling fewer or more items) will not result in a higher profit for the seller. As q_{it} can include zero, the decision of whether to list, as what number of items to list, is endogenous.

Equation 13 indicates whether a given seller will list q_{it} items. This decision is affected by the number of competing sellers and participating buyers. As the number of competing sellers increases, the total number of items listed to bidders can increase. Yet equation 6 places a constraint on the number of auctions in which a bidder can concurrently bid. In the presence of an increased number of items listed, this constraint lowers the likelihood that any given seller will be able to sell their items as an increase in the total listings leads to fewer bidders per auction listed. In the presence of a fixed listing fee this lowers the expected return from listing an item. Therefore, an increase in the number of sellers leads to a decrease in the expected number of items listed by each seller. One implication of this is that the auction house can manage this decrease in items listed per seller by lowering per item fees.

Likewise, the number of active buyers and their respective valuations affect the seller's listing decision. The expected return in equation 12 is a function of the second highest bid, as noted in equation 10. The second highest bid is, in turn, a function of the number of bidders J and their bidding thresholds x^* as indicated in equation 8. As more bidders become active (i.e., x^* decreases – which could occur if the mean of the valuation distribution increases) or the universe of bidders becomes larger (J increases – for example, the bidding population become larger) then the seller return increases and more sellers will list. In sum, the number of items listed affects buyer participation and the buyer participation affects the number of items listed. Hence, there is a structural link between seller and bidder behaviors.

Seller Acquisition Cost, C_{it}^s Given the foregoing concavity assumption, we specify the prior distribution of seller i 's acquisition cost to be:

$$C_{it}^s = \beta_{0i}^{C_s} BOOKVAL_{it}^{\beta_1^{C_s}} q_{it}^{\beta_2^{C_s}} + e_{q_{it}}^{C_s} \quad (14)$$

We include a seller-specific constant term, $\beta_{0i}^{C_s}$, to capture unobserved heterogeneity in seller costs. $e_{q_{it}}^{C_s}$ is an i.i.d. error term following normal distribution with mean 0 and variance θ^{-1} (the prior specifications for θ and other parameters are detailed in Appendix A3.1). The stochastic terms are known by the seller itself but unobserved by researchers. Delivery costs, which are unobserved, can affect both the average seller costs $\beta_{0i}^{C_s}$ and per item costs $\beta_2^{C_s}$ (Lewis et al. (2006)). These costs, which are unobserved, can affect both the average seller costs $\beta_{0i}^{C_s}$ and per item costs $\beta_2^{C_s}$. It is notable that observed variations in q_{it} over the duration of the data leads to estimated variations in the costs C_{it}^s over this duration. In turn, these variations in acquisition costs inform how sellers' listing behaviors vary with changes in costs. In this fashion we can extrapolate how other costs (such as changes in fees) affect listing behavior even if such changes are not observed in the data. Stated differently, though there is no variation in fees over time from which to impute the seller listing elasticity, we can use the link between acquisition costs and listings to assess how hypothetical changes in fees affect listings.

We adopt the following hierarchical structure for the heterogeneous constant term:

$$\begin{aligned} \beta_{0i}^{C_s} &\sim N\left(\bar{\beta}_0^{C_s}, (\phi^{C_s})^{-1}\right) \\ \bar{\beta}_0^{C_s} &\sim N(\bar{\beta}_0, \sigma_{0C_s}^2) \\ \phi^{C_s} &\sim \text{Gamma}(a_0^{C_s}, b_0^{C_s}) \end{aligned} \quad (15)$$

Similar to the bidder cost, multiple observations of sellers across auctions enable the identification of seller cost from valuation distribution (which is an important component determining seller revenues). In particular, seller costs are identified through the observed bids, bidder costs, and seller's listing behavior across multiple auctions.

4 Estimation

We adopt MCMC approach to estimate the model due to the flexibility of Bayesian methods (Rossi et al. (1996)). In particular, it enables us to readily accommodate latent variables via

data augmentation, including bidder valuations, bidder disutility and seller profits. Moreover, the approach enables us to readily estimate a complex model. In this section we define the likelihood and in the Appendix A3 we detail the sampling chain and priors.

4.1 The Conditional Likelihood of Bidder Model

Given the threshold level x_{ijt}^* , the conditional likelihood of bidder j bidding on auction itn is,

$$(p_{ijt}^{bid} | \mu_{it}, \sigma, MinBid_{it}, C_{ijt}^b) = [g(b_{ijt} | \mu_{it}, \sigma)]^{I^{bid}} [g(v_{ijt}^* | \mu_{it}, \sigma, MinBid_{it}, C_{ijt}^b)]^{1-I^{bid}} \quad (16)$$

where b_{ijt} is the bidder's revealed valuation from either a) the current auction n or b) the bidder's most recent bid on a like item (e.g., the same coin type) in a previous auction; I^{bid} equals one if the bidder's valuation has been revealed; v_{ijt}^* is a partially latent valuation drawn from the valuation distribution if $I^{bid} = 0$. The first term in (16) reflects the case where a bid is observed, and the likelihood of observing b_{ijt} on this instance is $g(b_{ijt} | \cdot)$. The second term reflects the case where a bid is not observed ($I^{bid} = 0$). In this case, the latent valuation v_{ijt}^* is obtained from the distribution $g(\cdot | \cdot)$ truncated at x_{ijt}^* , where x_{ijt}^* is defined in Theorem 1.

The product of equation 16 across ijt gives the conditional likelihood of bidder model,

$$L^{bidder} = \prod_{i,j,t,n} (p_{ijt}^{bid} | \cdot) \quad (17)$$

4.2 The Conditional Likelihood of Seller Model

The decision regarding the number of items to list, q_{it} , enters the likelihood as follows:

$$p_{it}^{list}(q_{observed} = q_{it} | \cdot) = \prod_{\tilde{q}_{it} \neq q_{it}} \Pr\{\pi_{it}(q_{it}) \geq \pi_{it}(\tilde{q}_{it})\} \quad (18)$$

That is, the estimated parameters should maximize the likelihood such that any deviations from the observed q_{it} can not result in extra profit. In practice, given the fact that \tilde{q}_{it} can assume an infinite number of values, it is impossible to calculate $\pi_{it}(\tilde{q}_{it} \neq q_{it})$ for all potential values of \tilde{q}_{it} . Instead, we approximate the likelihood by considering only those profits for a set of \tilde{q}_{it} 's that are within an arbitrarily large range around the observed q_{it} . Given that the profit function is assumed to be concave, any dramatic deviation of \tilde{q}_{it} from the observed q_{it} is unlikely to achieve a higher profit. Therefore the probability of $\Pr\{\pi_{it}(q_{it}) \geq \pi_{it}(\tilde{q}_{it} \neq q_{it})\}$ approaches 1. Hence, our approximation is not likely to be very restrictive.

The conditional likelihood of seller's model is the product of p_{it}^{list} across i, t ,

$$L^{seller} = \prod_{i,t} p_{it}^{list}(q_{observed} = q_{it}|\cdot) \quad (19)$$

Hence the conditional likelihood for the integrated bidder and seller model is

$$L = L^{bidder} \cdot L^{seller} \quad (20)$$

4.3 Augmented Full Posterior Distribution

By conditioning on the unobserved latent variables, v_{ijtn}^* , π_{it} and C_{ijtn}^b , the model likelihood can be written as follows,

$$\begin{aligned} L^{augmented}|\Omega &= \int_{C_{ijtn}^b} \int_{v_{ijtn}^*} \int_{\pi_{it}(q_{it})} \int_{\pi_{it}(\tilde{q}_{it})} (L^{bidder}|v_{ijtn}^*, C_{ijtn}^b)(L^{seller}|v_{ijtn}^*, C_{ijtn}^b, \pi_{it}(q_{it}), \pi_{it}(\tilde{q}_{it})) \\ &\quad \prod_{i,j,t,n} p(C_{ijtn}^b|\Omega_{C^b}) \prod_{i,j,t,n} p(v_{ijtn}^*|\Omega_{v^*}) \\ &\quad \prod_{i,t} p(\pi_{it}(q_{it})|\Omega_q) \prod_{i,t,\tilde{q}_{it}} p(\pi_{it}(\tilde{q}_{it})|\Omega_{\tilde{q}}) d\pi_{it}(\tilde{q}_{it}) d\pi_{it}(q_{it}) dv_{ijtn}^* dC_{ijtn}^b \end{aligned} \quad (21)$$

where L^{bidder} is the bidder model likelihood from equation 17, L^{seller} is the seller model likelihood from equation 19, C_{ijtn}^b is the latent bidder disutility from equation 6, v_{ijtn}^* is the latent bidder value from equation 1, $\pi_{it}(\tilde{q}_{it}|\cdot)$ and $\pi_{it}(q_{it}|\cdot)$ are latent profits from equation 12 and $\Omega = \{\Omega_{C^b}, \Omega_{v^*}, \Omega_q, \Omega_{\tilde{q}}\}$ are the model parameters. As the latent variables are unobserved, we must integrate over these variables to compute the likelihood. We perform this integration via an MCMC approach using data augmentation. One advantage of this approach is that it yields estimates for the distribution of the latent variables which become useful in our policy analysis (for example, we can estimate the unobserved listing costs for each seller and valuations of each bidder). Another advantage of this approach is that it facilitates the integration.

The augmented full posterior can then be constructed by multiplying the likelihood by the prior,

$$p(\Omega|L^{augmented}) \propto (L^{augmented}|\Omega) * p(\Omega)$$

where $p(\Omega)$ is the prior distribution for the model parameters. More details regarding the MCMC sampling chain and the choice of priors are presented in Appendix A3.

In Appendix A4, we present the design and results of a simulation that indicates our sampling chain is effective at recovering the true model parameters, enhancing our confidence in model

identification. All simulated parameters lie well within the 95% posterior predictive interval for these simulates.

5 Results

Table 6 reports the parameter estimates for the auctions model using the Celtic coin data. The Table indicates that book value and seller feedback have a positive effect on bidder valuations. For each \$1.00 increase in book value, on average bidders value the item an additional 90 cents. A \$1.00 increase in the minimum bid implies an 18-cent increase in bidder value and this may reflect the belief that a minimum bid is a signal of quality. Consistent with Ariely and Simonson (2003) and Reiley (2006) we find a positive correlation between minimum price and auction price; for each dollar increase in the seller’s minimum price, the bidder value increases \$0.18. There is no significant effect of a secret reserve on bidder valuations after controlling for the minimum bid. This null result may arise from the sellers’ infrequent use of this feature (in just over 2% of auctions) or because the presence of the secret reserve is not as informative as its level (which is not observed). Storefronts add 76 cents to the mean valuation of an item. The presence of a gallery leads to an increase in value of 54 cents while the presence of a subtitle increases average value by 56 cents. The fee charged to a seller for the gallery is 25 cents while the fee for the subtitle is 50 cents; both fees lie within the posterior 95% predictive interval suggesting that the pricing of these features is in line with the additional value they generate. In sum, item-specific, seller-specific characteristics of the auction and marketing all have material effects on bidders’ perceived valuations of the goods.

The number of auctions attended increases one’s bidding disutility, suggesting decreasing marginal returns for auctions. Meanwhile, consistent with an inter-temporal budget constraint, we find that a bidder’s likelihood of attending new auctions decreases if they won recently (or, alternatively, there are decreasing marginal returns for acquiring an additional item). A significant positive constant cost exists. The constant implies a fixed bidding disutility in the neighborhood of \$5.37. Since we use a log-log specification for bidder disutility, the parameters for *AttendedAuction* and *Lapse* can be interpreted as elasticities or sensitivities. Hence the bidding disutility elasticities for concurrent auctions attended and lapse of time are around 0.79 and -0.31 , respectively.

Acquisition opportunity costs for a seller increase with an item’s book value. In particular, given the log-log specification, one can interpret the parameters as elasticities. For a 1 percent increase in

Table 6: Posterior Means and Standard Deviations of Model Parameters

	Median	95% Interval
Valuation Distribution		
μ		
Constant	0.59*	(0.31, 0.82)
Book Value	0.90*	(0.89, 0.92)
(Seller Feedback)/100	0.03*	(0.02, 0.04)
Minimum Bid	0.18*	(0.15, 0.21)
Secret Reserve Dummy	0.28	(-0.16, 0.99)
Store Dummy	0.76*	(0.32, 1.21)
Gallery Picture Dummy	0.54*	(0.04, 0.79)
Subtitle Dummy	0.56*	(0.02, 2.21)
σ		
Constant	4.00*	(3.84, 4.41)
Bidder Disutility		
Number Auctions Attended	0.79*	(0.70, 0.95)
Lapse Since Last Winning	-0.31*	(-0.34, -0.28)
Mean Individual Constant	1.68*	(1.50, 1.78)
Seller Acquisition Cost		
Book Value	0.86*	(0.65, 1.07)
Number of Listings	1.27*	(1.02, 1.55)
Cost Error Variance	0.95*	(0.65, 1.19)
Mean Individual Constant	1.05*	(0.98, 1.13)
Log Marginal Likelihood	-21278.41	

the book value, there is an impact on the seller’s total cost in the amount of 0.86 percent. Further, the elasticity of seller cost for the number of listings is 1.27. The mean of the seller-specific constant term is 1.05. As the constant term serves as a scaling factor in our formulation of seller costs, the finding that this term is close to one suggests the scaling effect is immaterial. In other words, seller acquisition costs are largely determined by per item costs and the opportunity cost for multiple listings.

To illustrate the fit of our model, we simulate bid values within sample based on our estimates and data. We then compare the observed bid/book ratios with the simulated ratios. Figure 4 shows the comparison between the two variable distributions. The model fits the mean bid to book ratio very well (Observed = 0.48, Simulated = 0.48). The median is also comparable (0.31 vs. 0.39). Overall, the fit seems good, suggesting our model specification captures the observed bidding behaviors well.

Of note, we contrast the full model to one wherein we model only bidder behavior. The bidder-

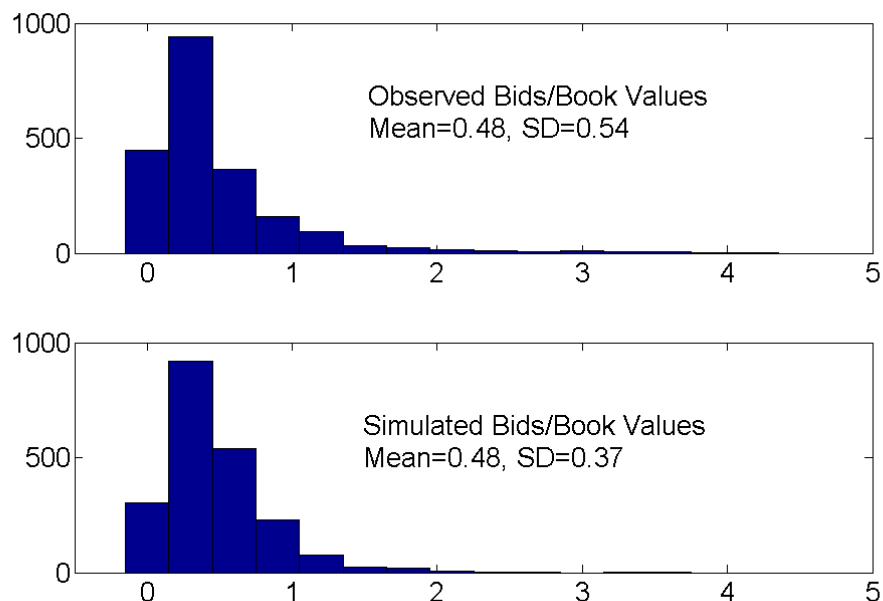


Figure 4: Observed Bids Vs. Simulated Bids

model parameter estimates are virtually identical to those obtained using the full model though the full model yields better predictions regarding bidder behavior, potentially suggesting gains in efficiency arising from joint estimation (the log-marginal likelihood for bidders in the joint model is -20062.5 vs. -20265.2 in the buyer only model). We speculate that the similarity in bidder parameter estimates across the bidder only and bidder-seller models reflects independence in errors across the bidder and lister models; this finding would be less likely to obtain in the face of omitted variables that could induce correlated errors. Though our results indicate joint estimation is more likely an issue of efficiency than bias, it is important to note that estimates pertaining to seller behavior are requisite for engaging the policy simulations that underpin the goals of this paper. Hence our research could not proceed without the seller model.

6 Managerial Implications

Whereas the parameter estimates are informative about bidder and seller behavior, they offer little explicit guidance to the auction house regarding its marketing strategy. We consider three strategic implications: a manipulation of auction house fees (which is informative about optimal pricing),

the attrition of a customer (which is informative about customer value), and the effect of seller characteristics on auction house revenues (which places a value on a seller’s reputational capital).

6.1 Auction House Pricing

6.1.1 Pricing Elasticity and Optimal Pricing

Using the model estimates, one can assess the effect of the auction house pricing strategy on the market equilibrium number of listings, bids and closing prices in the considered category.²¹ As noted above, we can infer the price-demand relationship even in the absence of any historic variation in fees because the structural model is informative about the latent bidder and seller costs and the bidder valuations. The price-demand relationship enables us to infer how a change in auction house fees affects seller profits, hence the seller listings and the resulting bidder demand. With information on the number and closing price of auctions, the auction house can assess how a fee change affects its revenue (which is equal to the total merchandise volume in the market times the percent commission plus the total number of listed items times the listing fee per item). In Table 7 we compute auction fee price elasticities by simulating the effect of a 1% decrease in current listing fees and commissions on profits.²²

Table 7: Fee and Commission Price Elasticities

Percentage Change	1% Decrease in Commissions	1% Decrease in Listing Fees
Seller Profit	3.90 (1.67, 5.90)	1.93 (0, 4.94)
Number of Auctions	2.04 (0, 8.95)	0.68 (0, 1.99)
Auction Website Revenue	0.86 (0.09, 1.85)	-0.08 (-0.11, -0.03)

Table 7 indicates that a decrease in fees improves seller profits, leading to more listings and more gross volume sold on the site.²³ However, the effect is much larger for commissions than listing fees.

²¹Note that our model assumes that buyers and seller behave optimally while our simulation presumes the auction house does not. Prima facie, this seems inconsistent. However, there exist some differences between these parties. First and foremost, the firm has indicated directly to us that its pricing does not follow an elasticity-based approach but that it is more heuristic based. Hence they are interested in using elasticity based pricing to inform their decisions, which is a key goal of this exercise. Second, buyers and sellers face repeated observations and play auctions over many occasions. This suggests the potential for learning and feedback, as such, the “as if” optimality assumption may be reasonable (Amaldoss and Jain (2005)). In contrast, little price variation exists for the auction house making it difficult to learn.

²²The auction house utilizes a tiered commission system. To protect the confidentiality of the data supplier, we do not reveal the actual commissions. To compute the elasticities, we decrease the commission at each level by 1%. Likewise we reduce the listing fees by 1%.

²³We do not report actual revenues to protect the confidentiality of the data provider. However, the actual revenue

As a result, the overall increase in volume at the auction site arising from a commission decrease generates enough total revenue to offset the reduction in per item commissions. In contrast, the increased volume arising from reduced per item listing fees does not offset the loss in the per item listing fee. All 95% posterior predictive intervals for these fee elasticities exclude zero. Accordingly, we would recommend that the auction house shift its fees for this category from commissions to listing fees by reducing the commissions and increasing the listing fees. At a minimum we would recommend that they refrain from raising commissions further by raising listing fees instead.

This recommendation can in part be explained by noting how different fee types affect sellers. Per item fees are an example of a uniform pricing strategy whereas commissions tend to disproportionately affect higher value items. As a result, a decrease in commissions disproportionately affects high value sellers relative to a decrease in fees. Simulating the effect of a 2 point decrease in commissions, we observe more high book value items listed and high feedback sellers listing more items relative to the case wherein per item fees are decreased. As higher book values and reputations lead to increased item valuations, a reduction in commissions disproportionately affects these groups thereby leading to an increase in high valuation listings. Given that these high valuation items generate higher profits for the seller, a small reduction in commissions can lead to a considerable increase in seller profits for such items and a concurrent increase in listings. We note that these effects are likely to be amplified in categories wherein valuations and feedback scores assume greater importance, such as art. To further assess the degree to which listing fees should be increased and commissions reduced for the considered category, we create a 9 by 6 grid of alternative commission and listing fee structures and estimate the house revenues associated with each fee combination. Table 8 reports the result of this analysis.

The upper left corner of Table 8 reflects the current level of listing fees and commissions. Each subsequent row indicates the effect of a one quarter point decrease in commissions. Thus, the last row indicates a two point decrease in commissions.²⁴ Each subsequent column from left to right indicates the effect of a 5% increase in the listing fee. Thus, the last column indicates a 25% increase in listing fees from the current level. The cells in Table 8 report the percentage change in auction house revenue for each combination of commission and listing fees; in general, revenues

lies within the 95% posterior predictive interval for estimated revenues.

²⁴We base the two point decrease upon the highest commission rate (which is associated with the lowest tier of closing prices). We then translate this point decrease to a percentage and apply it to the other pricing tiers.

Table 8: Revenue Percentage Changes with Alternative Pricing Schedules

Commissions (points)\Fees (%)	0	5	10	15	20	25
0	0	0.37	0.63	0.96	0.76	0.13
-0.25	1.04	1.48	1.74	1.77	1.21	0.30
-0.50	1.11	1.61	1.84	2.03	1.37	1.30
-0.75	1.19	1.87	2.19	2.89	2.66	1.76
-1.00	1.16	1.77	1.97	2.87	2.49	1.65
-1.25	1.12	1.38	1.68	1.90	1.61	1.33
-1.50	1.01	1.19	1.48	1.73	1.53	1.07
-1.75	0.97	1.18	1.48	1.71	1.52	0.81
-2.00	0.31	0.81	0.93	1.39	0.87	0.39

increase with lower commissions and higher fees. The maximum revenue increase of roughly 3% is associated with a 0.75 point reduction in commissions and a 15% increase in listing fees. Even were the auction house not to decrease commissions, it is possible to increase revenues by nearly 1% with a 15% increase in listing fees. To the extent similar increases could be realized across categories, a 3% increase could prove quite considerable.

6.1.2 Price Customization

Recently, Zhang and Wedel (2007) have proposed that the pricing customization across customers enhances profits in the context of frequently purchased grocery goods. We therefore seek to assess whether the result is similar in the context of auctions. One reason to believe customization may be efficacious is the relatively high dispersion in the value of items listed for sale in the auction context (book values range from \$3 to \$675). Sellers of high value listings (defined as those items with greater value than the median Book Value) stand to gain more from changes to commissions whereas sellers of low value items stand to gain more from changes in per item listing fees. To explore whether we can improve upon the pricing strategies in Table 8, we proceed as follows. Instead of increasing fees and lowering commissions to all sellers, we increase fees and lower commissions only for high value sellers. Given sellers of high value items are more sensitive to commissions and less sensitive to fees, targeting a price change to these sellers could prove profitable. Table 9 reports this analysis.

Comparing the first *columns* of Table 8 with those in Table 9, we observe that a 0.75 point

Table 9: Revenue Effects with Targeting of Commissions and Fees

Commissions (points)\Fees (%)	0	5	10	15	20	25
0	0	0.34	0.82	0.99	0.73	0.69
-0.25	1.00	1.37	1.70	1.98	1.26	1.28
-0.50	2.09	2.59	3.47	3.67	2.57	2.37
-0.75	2.93	3.13	3.82	3.91	2.96	2.47
-1.00	2.92	3.07	3.80	3.84	2.59	2.28
-1.25	2.21	2.49	2.62	2.31	2.19	2.00
-1.50	1.02	1.11	1.80	2.02	1.78	1.76
-1.75	0.91	1.03	1.73	1.81	1.91	1.16
-2.00	0.22	0.54	1.07	1.57	1.04	0.43

decrease in commissions (and no change in per item fees) targeted to only the high item value sellers yields a nearly 3.0% revenue increase in a customized setting but only a 1.2% increase in the uniform pricing setting. Not only does this triple the revenue gain observed in the non-targeted pricing setting, it exceeds the maximum potential return of 2.89% when prices are not customized. Therefore, targeting commissions seems to have a material effect on revenues. Contrasting the first rows of Table 8 to those in Table 9 yields a comparison between raising per item listing fees to all sellers with only raising fees to high value sellers (as low value sellers are presumably more sensitive to fees). As the cells are comparable across the two Tables, we conclude that there is little benefit to targeting fees relative to targeting commissions. Finally, Table 9 suggests the maximum return of the proposed customized strategy (obtained with a 0.75 point decrease in commission and a 15% increase in fee to high value sellers,) to be 3.91% compared to 2.89% from the blanket strategy. Thus, the potential gain arising from targeted pricing is about 35%. These numbers are in line with the findings of Zhang and Wedel (2007).

In sum, the joint model of bidder and seller behavior enables us to develop pricing prescriptions even in absence of any historical pricing variation. Given the total volume transacted in the area of Internet auctions, the pricing problem is an important one and the proposed model represents an initial step towards addressing this problem. In principle, other policy simulations are possible with our model, including an assessment of how a change in auction rules away from sealed bid second price auctions would affect auction house revenue.

6.2 Customer Value in a Two-sided Market

Valuing customers has seen increased research attention in marketing in recent years (Kamakura et al. (2005)). Firms seek to assess customer profitability in order to determine how much to invest in a given customer (for retention or acquisition), whether to divest an unprofitable customer and for firm valuation. Yet the customer valuation literature focuses on customer valuation in contexts where one customer's presence does not affect another. Valuing customers in two-sided markets is problematic because of the interaction between bidders and sellers. Exacerbating this consideration in the context of auctions is the role of competition in customer valuation. Should a seller attrite, the supply of auctions decreases. Given price increases are permanent, this has two enduring effects: i) fewer items listed leads to increased competition on the part of bidders for the remaining items, thereby driving up prices resulting in higher per-auction commissions for the auction house and ii) higher prices paid by buyers and lesser competition encourages the remaining sellers to list some additional items which will also generate revenue for the auction house. These factors, which we denote *indirect value effects*, will offset the initial revenue lost from a departing seller (denoted *direct value effects*) and these indirect effects could be considerable. A similar categorization applies on the bidder side. When one buyer departs the system, others that remain will bid for the items thereby offsetting this loss to some degree. However, softer competition among bidders could also lead to lower prices and therefore fewer items listed on the part of sellers. Thus, the direct value effect for a buyer in terms of auction revenue is offset to some degree by the presence of other bidders in the system provided this does not induce too many sellers to depart.

In the absence of a model that captures competitive interactions, it is unclear how to value customers. The most commonly used practice is to use the direct effect to measure customer value. However, this approach overstates the value of a customer by ignoring competition. Our solution to this problem is to conduct a policy experiment wherein a particular buyer or seller is excluded from the system and compute the change in the equilibrium number of listings and closing prices of those listings over the duration of the data. By comparing the new equilibrium to the original, we can compute the short-term value of a customer to the auction house. In doing so, we abstract away from long-term dynamics such as the growth (diffusion) of bidders and sellers (Gupta et al. (2006)). It is therefore important to delineate between our short-term measure of customer value (as in our approach) and the more forward-looking lifetime value metrics often used in marketing

(Gupta et al. (2006); Kamakura et al. (2005)). Table 10 reports the result of a simulation wherein the largest buyer and seller, respectively, attrite and the resulting loss in fees to the auction house.

Table 10: The Value of the Largest Customer

	Seller		Buyer	
	Dollars	Percent	Dollars	Percent
Direct Value	\$127	100%	\$26	100%
Indirect Value	-\$30	24%	-\$21	81%
Total Value	\$97	76%	\$5	19%

The first row of Table 10 presents the direct value effects or the revenue of the auctions in which the customer participates, reflecting the total fees arising from the auctions in which the largest seller and buyer transact (a commonly used metric for valuing a customer in a two-sided market). The third row reports the equilibrium revenue realized by the auction house when the customer attrites as computed by our model (we call this the *total effect* as it is the sum of the direct effect plus the indirect and opposite effect arising from competition). The second row reports the difference between the direct effect and the total effect (i.e., the indirect effect). We also compute the relative percentage of the total effect that arises from the direct and indirect effects. For example, the direct effect of the largest seller on auction house revenues is \$127. The revenue realized by the auction house were this seller to attrite is \$97 (the total effect). Thus, the direct effects overstate the total revenue loss by \$30, or nearly a third (\$30/\$97). Given that competition does not offset most of the loss, we conclude that products sold by this vendor in this market are not altogether highly substitutable with the other products being auctioned. A rather different conclusion is realized on the buyer side where the largest buyer generates \$26 in direct revenue but only \$5 in total revenue, suggesting the indirect effects to be \$21. When this buyer leaves the market, others fill the void. Accordingly, the direct effects overstate the total effect by over 400%.

As noted above, indirect effects manifest as a change in equilibrium prices and supply. We can apportion the indirect effects across prices and supply in order to ascertain which effect is greater. This decomposition yields insights into whether the largest customer is more likely to affect listings (driven by seller behavior) or the closing price per listing (driven by bidder behavior). Let N_0 denote the number of closing auctions listed by competing sellers prior to attrition of the largest seller and p_0 denote the $N_0 * 1$ vector of unit revenues (commission plus listing fees) of those

auctions. Similarly, we denote the number of auctions and vector of unit revenues after the largest seller attrites as N_s and p_s . Using this notation, we compute i) the indirect effects arising solely from a change in unit revenues holding the number of auctions fixed, $(\sum_{k=1}^{N_s} p_{sk} - \sum_{k=1}^{N_s} p_{0k})$ and ii) the expected change in revenue arising from a change in the number of auctions holding the unit revenues fixed, $(\sum_{k=1}^{N_s} p_{0k} - \sum_{k=1}^{N_0} p_{0k})$. We use a similar decomposition to assess the effect of buyer attrition. Table 11 presents the results of this analysis.

Table 11: Indirect Value Due to Listings and Prices

	Seller		Buyer	
	Value	Percent	Value	Percent
Indirect Value	\$30	100%	\$21	100%
Effect Due to Adjustment in Listings	\$12	39%	\$12	57%
Effect Due to Adjustment in Unit Revenues	\$18	61%	\$9	43%

Columns 2 and 3 report the results of this analysis for the attrition of the largest seller and columns 4 and 5 report the analogous results for the attrition of the largest buyer. Of the \$30 indirect competitive effects that offset the total effect of the largest seller's attrition, about 2/5 (\$12) of the competitive effects are due to other sellers listing more items and about 3/5 (\$18) of the competitive effect is due to higher prices that arise from bidders bidding on fewer goods on the market. Thus, seller attrition plays a major role on both competitor behavior (items listed) and bidder behavior (bids).

With regard to the departure of the largest buyer, about 2/5 (\$9) of the indirect effects can be attributed to the unit revenue change and 3/5 (\$12) can be accounted by changes of number of auctions. Although the biggest bidder attrites, other bidders step in to fill the void and generate \$21 to largely offset the \$26 loss. Overall we conclude that, under a two-sided market such as online auction, using direct revenue as the metric may overstate a customer's value considerably. To accurately measure the value of a customer, a firm needs to take into account the network externalities caused by the customer.

6.3 Elasticities of Seller Feedback Scores

As noted in the literature on signalling game, seller feedback should play a role in transmitting the quality of their product leading to increased bids (Cabral and Hortacsu (2006)). Table 6 confirms

this proposition suggesting an increase of 100 in the seller feedback score leads to an average increase in customer valuation of 3 cents. However, the magnitude of the feedback elasticity is unclear as a change in its value (holding other seller’s feedback scores fixed) will induce other sellers to change their listing behaviors, which in turn has a downstream influence on bidders. In Table 12 we report the effect of a 1% increase in seller feedback scores on the seller’s and auction house’s revenues.

Table 12: Seller Feedback Score Elasticity

Percentage Change	1% increase in Seller Feedback Scores
Seller Profit	0.15 (0.04, 0.19)
Number of Auctions	0.16 (0.06, 0.65)
Auction Website Revenue	0.12 (0.11, 0.13)

We find that a 1% increase in reputation leads to a 0.15% increase in seller profits assuming no competitive reaction in reputation. Using our estimates of cost, we can compute the break even value for increasing reputational capital. As our model estimates suggest that costs are roughly half of seller revenues, this calculation implies that if the cost of increasing reputation by 1% is less than 0.30% of revenue, then it is profitable to do so. In practice, the 0.30% threshold is an upper bound because it is reasonable to conjecture that competing sellers will also raise their reputation scores in response.

7 Conclusion

Internet auctions have grown exponentially over the past decade and are now a major world wide source of transactional volume. In spite of this ascendancy, few econometric models exist to explain the role of seller behavior in the context of auctions. Seller behaviors are of interest because a) these affect the optimal policy on the part of the auction house (such as the fee structure and auction design), b) seller behaviors affect the choices of bidders, which suggests the traditional assumption of exogenous seller behavior in the empirical IO literature may be questionable, and c) seller behavior is of interest in its own right (in assessing customer value, for example).

We redress this consideration via a structural model of seller and bidder behaviors across auctions. We posit sellers will list if the return from doing so exceeds their respective costs. Likewise, we presume the bidders will bid if the expected return from doing so exceeds their respective costs.

An important component of the model is its integration of bidder and seller behavior. Sellers, when deciding to list, must consider the bids likely to materialize if they list and also the potential of competitive listings. Likewise, bidders must consider the number of auctions across which they can bid. In this fashion, the behaviors of the two groups are linked. This linkage exemplifies a two-sided market wherein bidders and sellers interact on a common platform (Rochet and Tirole (2006)) and extends structural models of auctions into this domain.

Accordingly, we develop an empirical structural model which rationalizes bidder and seller behavior under the auction context. We estimate this model using six months of auction data for Celtic coins provided by an auction house. The data are consistent with an independent private value auction. We use MCMC techniques to estimate the costs and valuations that would be consistent with the bidding and listing behavior observed in the data. Once these are known, it becomes possible to assess the role of fees on equilibrium listings. That is, as fees rise, seller profits fall and they are less likely to list. Bidders, faced with fewer auctions, will increase their bids to win. The net effect can be determined in this fashion.

The estimates from our empirical model indicate that item characteristics (such as its book value), seller characteristics (their feedback scores), and marketing tools (minimum bid, the use of a store front, and additional item information revealed by a subtitle and a picture) have significant impacts on bidder valuations and thus their bids. We further develop insights into bidder and seller costs and valuations, such as the finding that a 1% increase in book value on average increases seller costs by 0.86%.

Using these results, our policy experiment indicates that an increase in listing fees will increase auction house revenues while an increase in commissions will decrease them. By searching over a grid of values in the neighborhood of its current fees, we find that the change in fees and commissions can bring an increase of nearly 3% in auction house revenues for the Celtic coin category. The intuition behind this result is that fees exemplify a uniform pricing policy whereas commissions are a form of high value pricing discrimination. As such, a decrease in commissions has a large effect on the profitability of high valuation items (such as those with a high book value and listed by high reputation sellers) leading to a large increase in listings for such items. Accordingly, we expect our finding that pricing should be reallocated toward fixed fees to be amplified in categories with more high valuations items and attenuated in categories with lower valuation items. We build further

upon this intuition by considering a targeted pricing strategy wherein high value users face lower commissions and higher per item fees than low value users. We find that customized pricing can increase revenues by 35% over the uniform pricing solution.

The approach developed herein is also useful for assessing short-term customer value in two sided markets. Customer value is relevant to firms seeking to assess how much to invest in retaining or acquiring customers and, to date, this literature has been silent on the value of customers in the context of two-sided markets. The primary information available to value a customer (such as a seller) is the revenue that customer generates for a firm. However, such a valuation fails to account for competitive effects such as the tendency of bidders to switch to other sellers when a seller attrites and the tendency of other sellers to list more items. In general, we find approaches that ignore such indirect effects to overstate customer value of the seller by nearly a third and the value of a buyer by over 400%. We further apportion these indirect seller effects into an increase in the number of listings by competing sellers who seek to capitalize on the departure of competition and the increase in prices arising from stiffer bidder competition for the remaining items. We find these effects to be roughly equal.

The complexity of the problem we consider necessitates some simplifications, which also represent both limits and future research opportunities. In particular, we approximate bidding dynamics using lagged purchases in the cost function to capture inter-temporal dynamics and multi-auction bidding that may be induced by an inter-temporal multi-item budget constraint or price expectations. A more complete specification regarding the dynamics might yield some novel insights. Further research is therefore warranted. Second, consistent with our data, our model utilizes the private value assumption to simplify the analysis. An extension to the common value setting would yield interesting insights regarding interactions among bidders. Third, we do not consider competition between auction houses. We believe this is a reasonable approach, as fees constitute a relatively small component of the overall cost of listing items in this category and because the large number of bidders and sellers on this site make it less likely that competing sites would be attractive alternatives to sellers. Nonetheless, inter-site competition is widely unaddressed in the literature and represents an important direction for future analysis. Fourth, we presume the auction house is not strategic in its price setting behavior. Though this assumption is predicated on conversations with price setters at this organization, many firms (e.g., Google and Yahoo!) are

strategic in their use of pricing mechanisms and it would be interesting to address this issue. Fifth, we consider listing features to be exogenous. It is unlikely that endogenizing these variables will yield a unique equilibrium, so it is unclear whether relaxing this restriction will be useful for policy analysis. Moreover, given the number of potential combinations of these features, it is not clear that it is possible to solve for all of them. Yet these features represent a source of revenue to the auction house, and the problem of how to price these is another area of importance to consider; recent advances in combinatorial optimization could prove fruitful here. In sum, we believe that the combination of the material economic importance of auctions and the large array of remaining problems for consideration suggests this is a rich area for future research. We hope our initial foray into the problem of bidder and seller networks in the context of auctions will lead to further research in this domain.

Appendix

A1 Proof of Theorem 1

In a second-price sealed bid auction with independent private values (IPV), the optimal equilibrium bidding strategy is for participants to bid their respective valuations (Milgrom and Weber (1982)). Thus, the second highest bid and the price paid by the winner to the seller is equivalent to the second highest valuation of participants (or the seller chosen minimum bid if the second highest bid is below the seller minimum). Accordingly, the winning bidder receives a return equal to their valuation less the price paid to the seller (i.e., the second highest bid or the minimum bid, whichever is greater) and less the bidder's latent cost of participating in the auction. Losing bidders receive no surplus and pay only their latent costs of participation. Bidder j 's expected return for participating in an auction is therefore given by

$$\begin{aligned} & E(\pi_{ijtn}) \\ &= \int_{-\infty}^{v_{ijtn}} (v_{ijtn} - \max(\alpha_{\bar{j}}, MinBid_{it})) f(\alpha_{\bar{j}}) d\alpha_{\bar{j}} - C_{ijtn}^b \end{aligned} \tag{A-1}$$

where $\alpha_{\bar{j}}$ is the highest rival's bid and $f(\alpha_{\bar{j}})$ is its density. The first term represents the expected revenue accruing to bidder j from participating in the auction, the second term is the expected payment to the seller and the third term is the bidding cost. Note that, when $\alpha_{\bar{j}} > v_{ijtn}$, bidder j loses the auction and receives no revenue and makes no payment to the seller. Hence the integral of $\alpha_{\bar{j}}$ in equation A-1 has an upper limit of v_{ijtn} . Bidder j will participate in the auction if and only if $E(\pi_{ijtn}) \geq 0$.

We claim that $E(\pi_{ijtn})$ is increasing in v_{ijtn} . This insight is necessary to establish the existence of a minimum bidding threshold below which a bidder will not participate in an auction and above which it becomes always profitable to do so. To show this, note that the partial derivative of

equation A-1 is²⁵

$$\begin{aligned}
& \frac{\partial E(\pi_{ijtn})}{\partial v_{ijtn}} \\
&= \int_{-\infty}^{v_{ijtn}} f(\alpha_{\bar{j}}) d\alpha_{\bar{j}} + v_{ijtn} \cdot f(v_{ijtn}) - v_{ijtn} \cdot f(v_{ijtn}) \\
&= \int_{-\infty}^{v_{ijtn}} f(\alpha_{\bar{j}}) d\alpha_{\bar{j}} > 0
\end{aligned} \tag{A-2}$$

As this expression is positive, a bidder's expected return always increases with their valuation. This implies that there exists a bidding threshold x_{ijtn}^* such that bidder j will enter the auction for any $v_{ijtn} \geq x_{ijtn}^*$ where x_{ijtn}^* is the root of $E(\pi_{ijtn}|\cdot) = 0$. Accordingly, one can obtain x_{ijtn}^* in the form of an implicit function by setting equation A-1 to zero:

$$\int_{-\infty}^{x_{ijtn}^*} (x_{ijtn}^* - \max(\alpha_{\bar{j}}, MinBid_{it})) f(\alpha_{\bar{j}}) d\alpha_{\bar{j}} - C_{ijtn}^b = 0 \tag{A-3}$$

Equation (A-3) is incumbent upon the distribution for $\alpha_{\bar{j}}$, which is given as follows:

$$\begin{aligned}
& f(\alpha_{\bar{j}}) \\
&= (J-1) \left\{ \int_{C_{ij'tn}^b} \{ [G(\alpha_{\bar{j}}|\mu_{it}, \sigma) - G(x_{ij'tn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha_{\bar{j}}) \right. \\
&\quad \left. + G(x_{ij'tn}^*|\mu_{it}, \sigma) \} f(C_{ij'tn}^b) dC_{ij'tn}^b \right\}^{J-2} \\
&\int_{C_{ij'tn}^b} g(\alpha_{\bar{j}}|\mu_{it}, \sigma) F_{x^*}(\alpha_{\bar{j}}) f(C_{ij'tn}^b) dC_{ij'tn}^b
\end{aligned} \tag{A-4}$$

where $F_{x^*}(\cdot)$ is the distribution of x^* . Equation A-4 can be explained as follows. Letting j' index the $J-1$ bidders excluding the focal bidder j ,²⁶ $\alpha_{\bar{j}}$ must be drawn from $\int_{C_{ij'tn}^b} g(\alpha_{\bar{j}}|\mu_{it}, \sigma) F_{x^*}(\alpha_{\bar{j}}) f(C_{ij'tn}^b) dC_{ij'tn}^b$ (the third row), i.e., the highest rival bidder has a value drawn from the distribution $g(\alpha_{\bar{j}}|\cdot)$ and the draw is higher than the threshold $x_{\alpha_{\bar{j}}}^*$ to be a positive bid. The integral over $C_{ij'tn}^b$ arises from bidder j 's uncertainty about other bidders' costs. Further, for $\alpha_{\bar{j}}$ to be the highest rival bid among the $J-1$ bidders, the remaining $J-2$ bidders (other than bidder j and the bidder with value $\alpha_{\bar{j}}$) must have values a) greater than their respective bidding thresholds $x_{ij'tn}^*$ but lower than $\alpha_{\bar{j}}$ or b) less than their respective bidding thresholds but higher than $\alpha_{\bar{j}}$, or c) less than their respective

²⁵Note that the formula for derivative with respect to the upper bound of an integral is

$$\frac{\partial \int_{x_1}^{x_2} f(t) dt}{\partial x_2} = f(x_2)$$

²⁶When there is a secret reserve price, the seller themselves can be considered as an additional "bidder".

bidding thresholds but lower than $\alpha_{\bar{j}}$. Condition a) gives the first row in the set of braces; condition b) and c) together give the second row in the set of braces. Hence we obtain the integral in the second row of equation A-4

$$\left\{ \int_{C_{ij'tn}^b} \{ [G(\alpha_{\bar{j}}|\mu_{it}, \sigma) - G(x_{ij'tn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha_{\bar{j}}) + G(x_{ij'tn}^*|\mu_{it}, \sigma) \} f(C_{ij'tn}^b) dC_{ij'tn}^b \right\}^{J-2}$$

where $F_{x^*}(\cdot)$ is the distribution of x^* . Because $x_{ij'tn}^*$ is define by an implicit function it is difficult to write this distribution in closed form. Hence we approximate this distribution by using the sample population distribution of $x_{ij'tn}^*$. This distribution can be obtained by first making draws from the distribution of $C_{ij'tn}^b$, then computing $x_{ij'tn}^*(C_{ij'tn}^b)$. The integral over $C_{ij'tn}^b$ again arises from bidder j 's uncertainty about other bidders' costs. Further, since any of the $J - 1$ bidders can offer the highest rival bid, we need to multiply the result by $\binom{J-1}{1} = J - 1$.

Note that bidders face a different distribution for $\alpha_{\bar{j}}$ (the highest competing bid) when making bidding decisions (equation A-4) than sellers face α (the second highest bid) when making listing decision (equation 8). This is mainly because the bidder calculates a first order statistics while the seller considers a second order statistics.

A2 Proof of Theorem 2

The second highest bid α has the distribution density $\int_{C_{ij'tn}^b} g(\alpha|\mu_{it}, \sigma, \alpha \geq x_{\alpha}^*) f(C_{ij'tn}^b) dC_{ij'tn}^b$, i.e., the bidder has a draw from the valuation distribution $g(\alpha|\cdot)$ and the draw is higher than the bidding threshold x_{α}^* . The integral over $C_{ij'tn}^b$ is due to the assumption that the seller only has knowledge regarding the distribution of bidders disutilities but is uncertain about the exact costs for each bidder. Therefore seller forms expectations about x_{α}^* by integrating over $C_{ij'tn}^b$. Further, for α to be the second highest bid, the following two constraints must be satisfied:

1. Among the J bidders, in addition to the top two highest bidders (the winning bid and the second highest bid, α), there remain $J - 2$ competitive bidders whose valuations are either a) higher than their bidding thresholds x^* but lower than α , or b) below their bidding thresholds, x^* ;
2. Among the J bidders, there exists exactly one bidder whose valuation exceeds both α and their bidding threshold, x^* .

The first constraint yields the first set of braces in Theorem 2:

$$\left\{ \int_{C_{ijtn}^b} \{ [G(\alpha|\mu_{it}, \sigma) - G(x_{ijtn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha) + G(x_{ijtn}^*|\mu_{it}, \sigma) \} f(C_{ijtn}^b) dC_{ijtn}^b \right\}^{J-2} \quad (\text{A-5})$$

where $F_{x^*}(\cdot)$ is the distribution of x_{ijtn}^* . Because x_{ijtn}^* is defined by an implicit function it is difficult to write this distribution in closed form. Hence we approximate this distribution by using the sample population distribution of x_{ijtn}^* . This distribution can be obtained by first making draws from the distribution of C_{ijtn}^b , then computing $x_{ijtn}^*(C_{ijtn}^b)$. Note that the two terms essentially “break up” the value distribution into a) those values that lie between α and x^* and b) those values that lie between x^* and the lower bound of the value distribution.

The second constraint leads to

$$1 - \int_{C_{ijtn}^b} \{ [G(\alpha|\mu_{it}, \sigma) - G(x_{ijtn}^*|\mu_{it}, \sigma)] F_{x^*}(\alpha) + G(x_{ijtn}^*|\mu_{it}, \sigma) \} f(C_{ijtn}^b) dC_{ijtn}^b$$

i.e., the complement of events represented by terms in the braces of equation A-5. This becomes the second set of braces in Theorem 2. Again, the integrals in both constraints are due to the randomness of C_{ijtn}^b . Further, since any of the J bidders can be the second highest bidder, we need to multiply the result by $\binom{J}{1} = J$. Similarly, any of the $(J - 1)$ bidders (other than the second highest one) can be the highest bidder, we also need to multiply the result by $\binom{J-1}{1} = J - 1$.

A3 MCMC Sampling Chain

A3.1 Priors

We run the sampling chain for 20,000 iterations; the first 10,000 iterations are used for burn-in and the remaining 10,000 are used for sampling. Based on inspection of the plots of the draws against iterations, we conclude the chain converges after around 4,000 iterations. The priors use a diffused variance of 100; examination of the final results shows that the choice of the variance is at least the order of magnitude greater than the variance of posterior distributions, which assures a proper but diffused prior (Spiegelhalter et al. (1996), Gelman et al. (2004)). We now detail the priors for the sampling chain.

	Priors	Selected Value
Valuation Distribution		
μ		
Constant, β_1^μ	$\beta_1^\mu \sim N\left(\mu_{\beta_1}, \sigma_{\beta_1}^2\right)$	$\mu_{\beta_1} = 0, \sigma_{\beta_1}^2 = 100$
Book Value, β_2^μ	$\beta_2^\mu \sim N\left(\mu_{\beta_2}, \sigma_{\beta_2}^2\right)$	$\mu_{\beta_2} = 1, \sigma_{\beta_2}^2 = 100$
Seller Feedback/100, β_3^μ	$\beta_3^\mu \sim N\left(\mu_{\beta_3}, \sigma_{\beta_3}^2\right)$	$\mu_{\beta_3} = 0, \sigma_{\beta_3}^2 = 100$
Minimum Bid, β_4^μ	$\beta_4^\mu \sim N\left(\mu_{\beta_4}, \sigma_{\beta_4}^2\right)$	$\mu_{\beta_4} = 0, \sigma_{\beta_4}^2 = 100$
Secret Reserve Dummy, β_5^μ	$\beta_5^\mu \sim N\left(\mu_{\beta_5}, \sigma_{\beta_5}^2\right)$	$\mu_{\beta_5} = 0, \sigma_{\beta_5}^2 = 100$
Online Store Dummy, β_6^μ	$\beta_6^\mu \sim N\left(\mu_{\beta_6}, \sigma_{\beta_6}^2\right)$	$\mu_{\beta_6} = 0, \sigma_{\beta_6}^2 = 100$
Gallery Picture Dummy, β_7^μ	$\beta_7^\mu \sim N\left(\mu_{\beta_7}, \sigma_{\beta_7}^2\right)$	$\mu_{\beta_7} = 0, \sigma_{\beta_7}^2 = 100$
Subtitle Dummy, β_8^μ	$\beta_8^\mu \sim N\left(\mu_{\beta_8}, \sigma_{\beta_8}^2\right)$	$\mu_{\beta_8} = 0, \sigma_{\beta_8}^2 = 100$
σ		
Constant	$\sigma \sim TN\left(\mu_\sigma, \sigma_{\sigma_\mu}^2\right)$	$\mu_\sigma = 1, \sigma_{\sigma_\mu}^2 = 100$
Bidder Disutility		
Bidder specific constant $\beta_{0j}^{C_b}$	$\beta_{0j}^{C_b} \sim N\left(\bar{\beta}_{0j}^{C_b}, (\phi^{C_b})^{-1}\right)$	
Hierarchical parameter $\bar{\beta}_0^{C_b}$	$\bar{\beta}_0^{C_b} \sim N(\bar{\beta}_0^{C_b}, \sigma_{0C_b}^2)$	$\bar{\beta}_{bidder}^{C_b} = 1, \sigma_{0C_b}^2 = 100$
Hierarchical parameter ϕ^{C_b}	$\phi^{C_b} \sim Gamma(a_0^{C_b}, b_0^{C_b})$	$a_0^{C_b} = 0.01, b_0^{C_b} = 100$
Number of Auctions Attended, $\beta_1^{C_b}$	$\beta_1^{C_b} \sim N\left(\mu_1^{C_b}, \sigma_{1C_b}^2\right)$	$\mu_1^{C_b} = 1, \sigma_{1C_b}^2 = 100$
Lapse since last winning, $\beta_2^{C_b}$	$\beta_2^{C_b} \sim N\left(\mu_2^{C_b}, \sigma_{2C_b}^2\right)$	$\mu_2^{C_b} = -0.5, \sigma_{2C_b}^2 = 100$
Seller Acquisition Cost		
Seller specific constant $\beta_{0i}^{C_s}$	$\beta_{0i}^{C_s} \sim N\left(\bar{\beta}_{0i}^{C_s}, (\phi^{C_s})^{-1}\right)$	
Hierarchical parameter $\bar{\beta}_0^{C_s}$	$\bar{\beta}_0^{C_s} \sim N(\bar{\beta}_0^{C_s}, \sigma_{0C_s}^2)$	$\bar{\beta}_0^{C_s} = 0, \sigma_{0C_s}^2 = 100$
Hierarchical parameter ϕ^{C_s}	$\phi^{C_s} \sim Gamma(a_0^{C_s}, b_0^{C_s})$	$a_0^{C_s} = 0.01, b_0^{C_s} = 100$
Book value, $\beta_1^{C_s}$	$\beta_1^{C_s} \sim N\left(\mu_1^{C_s}, \sigma_{1C_s}^2\right)$	$\mu_1^{C_s} = 1, \sigma_{1C_s}^2 = 100$
Number of Listings, $\beta_2^{C_s}$	$\beta_2^{C_s} \sim TN\left(\mu_2^{C_s}, \sigma_{2C_s}^2\right)$	$\mu_2^{C_s} = 3, \sigma_{2C_s}^2 = 100$
Cost Error Variance θ	$\theta \sim Gamma(a_0^\theta, b_0^\theta)$	$a_0^\theta = 0.01, b_0^\theta = 100$

A3.2 The Conditional Posterior for the Seller Acquisition Costs: $\beta_{0i}^{C_s}, \bar{\beta}_0^{C_s}, \phi^{C_s}, \beta_1^{C_s}, \beta_2^{C_s}, \theta$ and π_{it}

- Let $X_{qit}(q_{it}) = BOOKVAL_{it}^{\beta_1^{C_s}} q_{it}^{\beta_2^{C_s}}$, $Y_{qit}(q_{it}) = q_{it} \cdot (1 - commission)E(R_{itn}^s | \cdot) - q_{it} \cdot fee_{it}$ and $\epsilon_{qit} = -e_{qit}^{C_s}$. Then equation (12) can be written as

$$\begin{aligned}
\pi_{it}(q_{it}) &= q_{it} \cdot (1 - commission)E(R_{itn}^s | \cdot) - C_{it}^s(q_{it}) - q_{it} \cdot fee_{it} & (A-6) \\
&= Y_{qit}(q_{it}) - \beta_{0i}^{C_s} X_{qit}(q_{it}) + \epsilon_{qit}
\end{aligned}$$

Similarly, for a deviation from q_{it} , the profit would be

$$\begin{aligned}\pi_{it}(\tilde{q}_{it}) &= \tilde{q}_{it} \cdot (1 - \text{commission})E(R_{itn}^s|\cdot) - C_{it}^s(\tilde{q}_{it}) - \tilde{q}_{it} \cdot fee_{it} \\ &= Y_{\tilde{q}_{it}}(\tilde{q}_{it}) - \beta_{0i}^{C_s} X_{\tilde{q}_{it}}(\tilde{q}_{it}) + \epsilon_{\tilde{q}_{it}}\end{aligned}\quad (\text{A-7})$$

For each iteration through the sampling chain, a set of latent $\pi(q_{it})$ will be drawn from a normal distribution with mean $Y_{q_{it}}(q_{it}) - \beta_{0i}^{C_s} X_{q_{it}}(q_{it})$ and variance $(\theta)^{-1}$. A set of deviated profits $\pi(\tilde{q}_{it})$ will be drawn from a truncated normal distribution with mean $Y_{\tilde{q}_{it}}(\tilde{q}_{it}) - \beta_{0i}^{C_s} X_{\tilde{q}_{it}}(\tilde{q}_{it})$, variance $(\theta)^{-1}$ and right truncation at $\pi(q_{it})$. We consider 20 such \tilde{q}_{it} and have found results to be robust to this choice.

- $\beta_{0i}^{C_s}$

$$\text{Prior } \beta_{0i}^{C_s} \sim N(\bar{\beta}_0^{C_s}, (\phi^{C_s})^{-1}) \quad (\text{A-8})$$

$$\begin{aligned}\text{Likelihood } L &\propto \prod_{T_i, \tilde{q}_{it} \neq q_{it}} \exp\left(-\frac{[\pi_{it}(\tilde{q}_{it}) - (Y_{\tilde{q}_{it}} - \beta_{0i}^{C_s} X_{\tilde{q}_{it}})]^2 \theta}{2}\right) \\ &\quad \prod_{T_i, q_{it}} \exp\left(-\frac{[\pi_{it}(q_{it}) - (Y_{q_{it}} - \beta_{0i}^{C_s} X_{q_{it}})]^2 \theta}{2}\right) \\ &\quad \exp\left(-\frac{(\beta_{0i}^{C_s} - \bar{\beta}_0^{C_s})^2 \phi^{C_s}}{2}\right)\end{aligned}\quad (\text{A-9})$$

$$\text{Posterior } (\beta_{0i}^{C_s}|\cdot) \sim N(\hat{\mu}_i^{C_s}, \hat{\Sigma}_i^{C_s}) \quad (\text{A-10})$$

$$\hat{\Sigma}_i^{C_s} = [(X_{q_i, \tilde{q}_i})' (X_{q_i, \tilde{q}_i}) \theta + \phi^{C_s}]^{-1} \quad (\text{A-11})$$

$$\hat{\mu}_i^{C_s} = \hat{\Sigma}_i^{C_s} [(X_{q_i, \tilde{q}_i})' (\pi_i(q_i, \tilde{q}_i) - Y_{q_i, \tilde{q}_i}) \theta + \bar{\beta}_0^{C_s} \phi^{C_s}] \quad (\text{A-12})$$

where T_i is the total number of items listed by seller i ; (X_{q_i, \tilde{q}_i}) is the vector containing $X_{q_{it}}$ and $X_{\tilde{q}_{it}}$ of seller i ; $(\pi_i(q_i, \tilde{q}_i) - Y_{q_i, \tilde{q}_i})$ is a vector containing all $(\pi_{it}(q_{it}) - Y_{q_{it}})$ and $(\pi_{it}(\tilde{q}_{it}) - Y_{\tilde{q}_{it}})$ of seller i .

- $\bar{\beta}_0^{C_s}$

$$\text{Prior } \bar{\beta}_0^{C_s} \sim N(\bar{\beta}_0^{C_s}, \sigma_{0C_s}^2) \quad (\text{A-13})$$

$$\text{Likelihood } L \propto \prod_{i=1}^I \exp\left(-\frac{(\beta_{0i}^{C_s} - \bar{\beta}_0^{C_s})^2 \phi^{C_s}}{2}\right) \quad (\text{A-14})$$

$$\text{Posterior } (\bar{\beta}_0^{C_s} | \cdot) \sim N(\lambda^{C_s}, \Lambda^{C_s}) \quad (\text{A-15})$$

$$\Lambda^{C_s} = [\sigma_{0C_s}^{-2} + \phi^{C_s}]^{-1} \quad (\text{A-16})$$

$$\lambda^{C_s} = \Lambda^{C_s} [\sum_i \beta_{0i}^{C_s} \phi^{C_s} + \bar{\beta}_0^{C_s} / \sigma_{0C_s}^2] \quad (\text{A-17})$$

where I is the total number of sellers observed.

- ϕ^{C_s}

$$\text{Prior } \phi^{C_s} \sim \text{Gamma}(a_0^{C_s}, b_0^{C_s}) \quad (\text{A-18})$$

$$\text{Likelihood } L \propto \prod_{i=1}^I (\phi^{C_s})^{1/2} \exp\left(-\frac{(\beta_{0i}^{C_s} - \bar{\beta}_0^{C_s})^2 \phi^{C_s}}{2}\right) \quad (\text{A-19})$$

$$\text{Posterior } (\phi^{C_s} | \cdot) \sim \text{Gamma}(\tilde{a}^{C_s}, \tilde{b}^{C_s}) \quad (\text{A-20})$$

$$\tilde{a}^{C_s} = I/2 + a_0^{C_s} \quad (\text{A-21})$$

$$\tilde{b}^{C_s} = 2b_0^{C_s} [2 + b_0^{C_s} \cdot s]^{-1} \quad (\text{A-22})$$

$$s = \sum_{i=1}^I (\beta_{0i}^{C_s} - \bar{\beta}_0^{C_s})^2 \quad (\text{A-23})$$

- An updated $\beta_1^{C_s(k)}$ in the k -th iteration is obtained with a random walk proposal density. For this we use a normal distribution with mean of $\beta_1^{C_s(k-1)}$ from last iteration, variance $\sigma_{\beta_1^{C_s}}$. The value of $\sigma_{\beta_1^{C_s}}$ is determined such that the acceptance rate of proposed values is between 15%-50% (Roberts (1996)). The proposed $\beta_1^{C_s(k)}$ is accepted with the probability of $\kappa^* = \min\{1, \kappa\}$, where

$$\kappa = \frac{L(\beta_1^{C_s(k)} | \cdot) p(\beta_1^{C_s(k)} | \mu_1^{C_s}, \sigma_{1C_s}^2)}{L(\beta_1^{C_s(k-1)} | \cdot) p(\beta_1^{C_s(k-1)} | \mu_1^{C_s}, \sigma_{1C_s}^2)} \quad (\text{A-24})$$

$L(\beta_1^{C_s(\cdot)}|\cdot)$ denotes the conditional likelihood and

$$L(\beta_1^{C_s(\cdot)}|\cdot) \propto \prod_{t,i,\tilde{q}_{it} \neq q_{it}} \exp\left(-\frac{\left[\pi_{it}(\tilde{q}_{it}) - \left(Y_{\tilde{q}_{it}} - \beta_{0i}^{C_s} X_{\tilde{q}_{it}}\right)\right]^2 \theta}{2}\right) \quad (\text{A-25})$$

$$\prod_{t,i} \exp\left(-\frac{\left[\pi_{it}(q_{it}) - \left(Y_{q_{it}} - \beta_{0i}^{C_s} X_{q_{it}}\right)\right]^2 \theta}{2}\right) \quad (\text{A-26})$$

$p\left(\beta_1^{C_s(\cdot)}|\mu_1^{C_s}, \sigma_{1C_s}^2\right)$ is the prior density evaluated at $\beta_1^{C_s(\cdot)}$.

- An updated $\beta_2^{C_s(k)}$ in the k -th iteration is obtained with a random walk proposal density. For this we use a truncated normal distribution with mean of $\beta_2^{C_s(k-1)}$ from last iteration, variance $\sigma_{\beta_2^{C_s}}$ and left truncated at 1. The value of $\sigma_{\beta_2^{C_s}}$ is determined such that the acceptance rate of proposed values is between 15%-50% (Roberts (1996)). The proposed $\beta_2^{C_s(k)}$ is accepted with the probability of $\kappa^* = \min\{1, \kappa\}$, where

$$\kappa = \frac{L(\beta_2^{C_s(k)}|\cdot)p\left(\beta_2^{C_s(k)}|\mu_2^{C_s}, \sigma_{2C_s}^2\right)\rho(\beta_2^{C_s(k-1)}|\beta_2^{C_s(k)}, \sigma_{\beta_2^{C_s}})}{L(\beta_2^{C_s(k-1)}|\cdot)p\left(\beta_2^{C_s(k-1)}|\mu_2^{C_s}, \sigma_{2C_s}^2\right)\rho(\beta_2^{C_s(k)}|\beta_2^{C_s(k-1)}, \sigma_{\beta_2^{C_s}})} \quad (\text{A-27})$$

where $L(\beta_2^{C_s(\cdot)}|\cdot)$ denotes the conditional likelihood and

$$L(\beta_2^{C_s(\cdot)}|\cdot) \propto \prod_{t,i,\tilde{q}_{it} \neq q_{it}} \exp\left(-\frac{\left[\pi_{it}(\tilde{q}_{it}) - \left(Y_{\tilde{q}_{it}} - \beta_{0i}^{C_s} X_{\tilde{q}_{it}}\right)\right]^2 \theta}{2}\right) \quad (\text{A-28})$$

$$\prod_{t,i} \exp\left(-\frac{\left[\pi_{it}(q_{it}) - \left(Y_{q_{it}} - \beta_{0i}^{C_s} X_{q_{it}}\right)\right]^2 \theta}{2}\right) \quad (\text{A-29})$$

$p\left(\beta_2^{C_s(\cdot)}|\mu_2^{C_s}, \sigma_{2C_s}^2\right)$ is the prior density evaluated at $\beta_2^{C_s(\cdot)}$.

$\rho(\cdot|\cdot)$ represents the density of the proposal distribution evaluated with $\beta_2^{C_s(k)}, \beta_2^{C_s(k-1)}, \sigma_{\beta_2^{C_s}}$ as input and parameters. For example, $\rho(\beta_2^{C_s(k-1)}|\beta_2^{C_s(k)}, \sigma_{\beta_2^{C_s}})$ is the density evaluated at $\beta_2^{C_s(k-1)}$ with $\beta_2^{C_s(k)}$ and $\sigma_{\beta_2^{C_s}}$ as mean and variance, respectively. Note that since the proposal distribution is truncated at 1 and thus asymmetric, the ratio between the two densities is used as a weight to obtain the correct acceptance probability.

- θ

$$\text{Prior } \theta \sim \text{Gamma}(a_0^\theta, b_0^\theta) \quad (\text{A-30})$$

$$\text{Likelihood } L \propto \prod_{i,t,\tilde{q}_{it} \neq q_{it}} (\theta)^{1/2} \exp \left(- \frac{[\pi_{it}(\tilde{q}_{it}) - (Y_{\tilde{q}_{it}} - \beta_{0i}^{C_s} X_{\tilde{q}_{it}})]^2 \theta}{2} \right) \quad (\text{A-31})$$

$$\text{Posterior } (\theta|\cdot) \sim \text{Gamma}(\tilde{a}^\pi, b_0^\theta) \quad (\text{A-32})$$

$$\tilde{a}^\pi = IT(\tilde{Q} + 1) / 2 + a_0^\theta \quad (\text{A-33})$$

$$b_0^\theta = 2b_0^\theta [2 + b_0^\theta \cdot s]^{-1} \quad (\text{A-34})$$

$$s = \sum_{i,t,\tilde{q}_{it} \neq q_{it}} [\pi_{it}(\tilde{q}_{it}) - (Y_{\tilde{q}_{it}} - \beta_{0i}^{C_s} X_{\tilde{q}_{it}})]^2 \quad (\text{A-35})$$

$$+ \sum_{i,t} [\pi_{it}(q_{it}) - (Y_{q_{it}} - \beta_{0i}^{C_s} X_{q_{it}})]^2 \quad (\text{A-36})$$

A3.3 The Conditional Posterior for the Bidder Valuation Model, $\beta^\mu, \mu, \sigma, v_{itn}^* | I^{bid}$

- β^μ

$$\text{Prior: } \beta^\mu \sim N(\mu_{\beta^\mu}, \Sigma_{\beta^\mu}) \quad (\text{A-37})$$

$$\text{Likelihood } L \propto \prod_{i,t,n} \exp \left(- \frac{(\mu_{it} - Z_{it}^\mu \beta^\mu)^2}{2} \right) \quad (\text{A-38})$$

$$\text{Posterior: } (\beta^\mu|\cdot) \sim N(\hat{\mu}_{\beta^\mu}, \hat{\Sigma}_{\beta^\mu}) \quad (\text{A-39})$$

$$\text{with } \hat{\Sigma}_{\beta^\mu} = \{\Sigma_{\beta^\mu}^{-1} + Z^{\mu'} \cdot Z^\mu\}^{-1} \quad (\text{A-40})$$

$$\hat{\mu}_{\beta^\mu} = \hat{\Sigma}_{\beta^\mu} \{Z^{\mu'} \cdot \mu + \Sigma_{\beta^\mu}^{-1} \mu_{\beta^\mu}\} \quad (\text{A-41})$$

where μ_{β^μ} is a column vector of dimension 8 containing all prior means for β^μ ; Σ_{β^μ} is a 8×8 matrix with the prior variances of β^μ on the diagonal and 0 as off-diagonal elements; μ is a column vector of the μ_{it} across itn with a dimension of ITN ; Z^μ is a the matrix in the dimension of $ITN \times 8$, containing independent variables determining the valuation distribution mean, μ .

- $v_{ijtn}^* | I^{bid}$

For observations where $I^{bid} = 0$ (see Section (4.1)), Theorem 1 indicates that v_{ijtn}^* is drawn from a right truncated $N(\mu_{it}, \sigma)$ with truncation level x_{ijtn}^* . This becomes the v_{ijtn}^* for all subsequent items of the same type.

- μ and σ

There is no closed form density for the conditional posterior of μ_{it} so we use a random walk Metropolis algorithm to obtain updated draws through the MCMC chain. A normal distribution is used as the proposal density. The proposal density has mean $\mu_{it}^{(k-1)}$ from $(k-1)$ -th iteration and a variance of Σ_μ . The value of Σ_μ is determined such that the acceptance rate of proposed values is between 15%-50%. With the proposal density, an updated value is generated and denoted as $\mu_{it}^{(k)}$.

The proposed $\mu_{it}^{(k)}$ is accepted with the probability κ^* , where

$$\begin{aligned} \kappa^* &= \min\{1, \kappa\} \\ \kappa &= \frac{L(\mu_{it}^{(k)}|\cdot)}{L(\mu_{it}^{(k-1)}|\cdot)} \end{aligned} \quad (\text{A-42})$$

where the conditional likelihood

$$L(\mu_{it}^{(\cdot)}|\cdot) \propto L^{bidder} L^{seller} p(\mu_{it} | Z_{it}^\mu, \beta^\mu) \quad (\text{A-43})$$

$L(\mu_{it}^{(k)}|\cdot)$ denotes the conditional likelihood evaluated at $\mu_{it}^{(k)}$ and $L(\mu_{it}^{(k-1)}|\cdot)$ is the analogy.

For the sampling of an updated σ , a truncated normal distribution is used as the proposal density as the variance must be positive. The proposal density has mean $\sigma^{(k-1)}$, variance Σ_σ and left truncated at zero.

$$\begin{aligned} \kappa^* &= \min\{1, \kappa\} \\ \kappa &= \frac{L(\sigma^{(k)}|\cdot) p\left(\sigma^{(k)} | \mu_\sigma, \sigma_{\sigma_\mu}^2\right) \rho(\sigma^{(k-1)} | \sigma^{(k)}, \Sigma_\sigma)}{L(\sigma^{(k-1)}|\cdot) p\left(\sigma^{(k-1)} | \mu_\sigma, \sigma_{\sigma_\mu}^2\right) \rho(\sigma^{(k)} | \sigma^{(k-1)}, \Sigma_\sigma)} \end{aligned} \quad (\text{A-44})$$

where the conditional likelihood

$$L(\sigma^{(\cdot)}|\cdot) \propto L^{bidder} L^{seller} \quad (\text{A-45})$$

$L(\sigma^{(k)}|\cdot)$ is the posterior likelihood evaluated at $\sigma^{(k)}$ and $L(\sigma^{(k-1)}|\cdot)$ is the analog for $\sigma^{(k-1)}$. $p\left(\sigma^{(k)}|\mu_\sigma, \sigma_{\sigma_\mu}^2\right)$ is the prior density evaluated at $\sigma^{(k)}$ and $p\left(\sigma^{(k-1)}|\mu_\sigma, \sigma_{\sigma_\mu}^2\right)$ is the analog for $\sigma^{(k-1)}$. $\rho(\cdot|\cdot)$ represents the density of the proposal distribution evaluated with $\sigma^{(k)}$, $\sigma^{(k-1)}$ and Σ_σ as input and parameters. For example, $\rho(\sigma^{(k-1)}|\sigma^{(k)}, \Sigma_\sigma)$ is the density evaluated at $\sigma^{(k-1)}$ with $\sigma^{(k)}$ and Σ_σ as mean and variance, respectively. Note that since the proposal distribution is truncated at zero and thus asymmetric, the ratio between the two densities is used as a weight to obtain the correct acceptance probability.

A3.4 The Conditional Posterior for the Bidder Cost Model, $\beta_{0j}^{C_b}$, $\bar{\beta}_0^{C_b}$, ϕ^{C_b} , β^{C_b} and C_{ijtn}^b

- $\beta_{0j}^{C_b}$

$$\text{Prior } \beta_{0j}^{C_b} \sim N\left(\bar{\beta}_0^{C_b}, (\phi^{C_b})^{-1}\right) \quad (\text{A-46})$$

$$\text{Likelihood } L \propto \prod_{itn} \exp\left(-\frac{(\log C_{ijtn}^b - \beta_{0j}^{C_b} - Z_{ijtn}^{C_b} \beta^{C_b})^2}{2}\right) \quad (\text{A-47})$$

$$\text{Posterior } (\beta_{0j}^{C_b}|\cdot) \sim N(\hat{\mu}_j^{C_b}, \hat{\Sigma}_j^{C_b}) \quad (\text{A-48})$$

$$\hat{\Sigma}_j^{C_b} = [ITN + \phi^{C_b}]^{-1} \quad (\text{A-49})$$

$$\hat{\mu}_j^{C_b} = \hat{\Sigma}_j^{C_b} [1'_{ITN} (C_j^b - Z_j^{C_b} \beta^{C_b}) + \bar{\beta}_0^{C_b} \phi^{C_b}] \quad (\text{A-50})$$

where ITN is the total number of auctions across itn ; C_j^b is the vector of the latent (log) costs of bidder j , having a length of ITN ; $Z_j^{C_b}$ is an $2 * ITN$ matrix containing *AttendedAuction* and *Lapse*; β^{C_b} is the vector of parameters associated with $Z_j^{C_b}$; $1'_{ITN}$ is a vector of 1's with the length of ITN ; and $var(\log C_{ijtn}^b) = 1$.

- $\bar{\beta}_0^{C_b}$

$$\text{Prior } \bar{\beta}_0^{C_b} \sim N(\bar{\beta}_0^{C_b}, \sigma_{0C_b}^2) \quad (\text{A-51})$$

$$\text{Likelihood } L \propto \prod_{j=1}^J \exp\left(-\frac{(\beta_{0j}^{C_b} - \bar{\beta}_0^{C_b})^2 \phi^{C_b}}{2}\right) \quad (\text{A-52})$$

$$\text{Posterior } (\bar{\beta}_0^{C_b} | \cdot) \sim N(\lambda^{C_b}, \Lambda^{C_b}) \quad (\text{A-53})$$

$$\Lambda^{C_b} = [\sigma_{0C_b}^{-2} + \phi^{C_b}]^{-1} \quad (\text{A-54})$$

$$\lambda^{C_b} = \Lambda^{C_b} \left[\sum_j \beta_{0j}^{C_b} \phi^{C_b} + \bar{\beta}_0^{C_b} / \sigma_{0C_b}^2 \right] \quad (\text{A-55})$$

where J is the total number of bidders observed.

- ϕ^{C_b}

$$\text{Prior } \phi^{C_b} \sim \text{Gamma}(a_0^{C_b}, b_0^{C_b}) \quad (\text{A-56})$$

$$\text{Likelihood } L \propto \prod_{j=1}^J (\phi^{C_b})^{1/2} \exp\left(-\frac{(\beta_{0j}^{C_b} - \bar{\beta}_0^{C_b})^2 \phi^{C_b}}{2}\right) \quad (\text{A-57})$$

$$\text{Posterior } (\phi^{C_b} | \cdot) \sim \text{Gamma}(\tilde{a}^{C_b}, \tilde{b}^{C_b}) \quad (\text{A-58})$$

$$\tilde{a}^{C_b} = J/2 + a_0^{C_b} \quad (\text{A-59})$$

$$\tilde{b}^{C_b} = 2b_0^{C_b} [2 + b_0^{C_b} \cdot s]^{-1} \quad (\text{A-60})$$

$$s = \sum_{j=1}^J (\beta_{0j}^{C_b} - \bar{\beta}_0^{C_b})^2 \quad (\text{A-61})$$

- β^{C_b}

$$\text{Prior: } \beta^{C_b} \sim N(\mu^{C_b}, \Sigma_{C_b}) \quad (\text{A-62})$$

$$\text{Likelihood } L \propto \prod_{ijtn} \exp\left(-\frac{(\log C_{ijtn}^{C_b} - \beta_{0j}^{C_b} - Z_{ijtn}^{C_b} \beta^{C_b})^2}{2}\right) \quad (\text{A-63})$$

$$\text{Posterior: } (\beta^{C_b} | \cdot) \sim N(\hat{\mu}_{\beta^{C_b}}, \hat{\Sigma}_{\beta^{C_b}}) \quad (\text{A-64})$$

$$\text{with } \hat{\Sigma}_{\beta^{C_b}} = \{\Sigma_{C_b}^{-1} + Z^{C_b'} \cdot Z^{C_b}\}^{-1} \quad (\text{A-65})$$

$$\hat{\mu}_{\beta^{C_b}} = \hat{\Sigma}_{\beta^{C_b}} \{Z^{C_b'} \cdot (C^b - \beta_0^{C_b}) + \Sigma_{C_b}^{-1} \times \mu^{C_b}\} \quad (\text{A-66})$$

where Z^{C_b} is a matrix containing all the $Z_{ijtn}^{C_b}$ across $ijtn$ and is dimensioned $IJTN$ by 2; C^b is a column vector containing all log bidder costs across $ijtn$ with a length of $IJTN$; $\beta_0^{C_b}$ is a column vector with a length of $IJTN$ (where $\beta_0^{C_b}$ can be construed as J stacked sub-vectors where the j -th sub-vector has dimension ITN and $\beta_{0j}^{C_b}$ as its elements); μ^{C_b} is a 2 by 1 vector of whose elements contain the prior means for parameters corresponding to Z^{C_b} (*AttendedAuction* and *Lapse*); and Σ_{C_b} is a 2 by 2 matrix whose diagonal elements contain the prior variances and whose off-diagonals are zeroes.

- C_{ijtn}^b

An updated $C_{ijtn}^{b(k)}$ in the k -th iteration is obtained with a random walk proposal density with mean $C_{ijtn}^{b(k)}$ from iteration $k - 1$ and variance σ_{C^b} . The candidate draw $C_{ijtn}^{b(k)}$ is accepted with the probability of $\kappa^* = \min\{1, \kappa\}$, where

$$\kappa = \frac{L(C_{ijtn}^{b(k)}|\cdot)\rho(C_{ijtn}^{b(k-1)}|C_{ijtn}^{b(k)}, \sigma_{C^b})}{L(C_{ijtn}^{b(k-1)}|\cdot)\rho(C_{ijtn}^{b(k)}|C_{ijtn}^{b(k-1)}, \sigma_{C^b})} \quad (\text{A-67})$$

where $L(C_{ijtn}^{b(\cdot)}|\cdot)$ denotes the conditional likelihood and

$$L(C_{ijtn}^{b(\cdot)}|\cdot) \propto L^{bidder} L^{seller} p(C_{ijtn}^b | \beta_{0j}^{C_b}, Z_{ijtn}^{C_b}, \beta^{C_b}) \quad (\text{A-68})$$

Thus, $L(C_{ijtn}^{b(\cdot)}|\cdot)$ is the conditional likelihood evaluated at $C_{ijtn}^{b(k)}$ and $L(C_{ijtn}^{b(k-1)}|\cdot)$ is the analog at iteration $k - 1$.²⁷

$\rho(\cdot)$ connotes the density of the proposal distribution evaluated with $C_{ijtn}^{b(k)}$, $C_{ijtn}^{b(k-1)}$, and σ_{C^b} as inputs. As the proposal distribution is truncated and asymmetric, the ratio between the two densities of $\rho(C_{ijtn}^{b(k-1)}|C_{ijtn}^{b(k)}, \sigma_{C^b})$ and $\rho(C_{ijtn}^{b(k)}|C_{ijtn}^{b(k-1)}, \sigma_{C^b})$ is used as a weight to obtain the correct acceptance probability.

A4 Monte Carlo Simulation

We develop a simulated data set designed to reflect our model and data in order to assess whether the proposed estimation approach can recover true parameters for an arbitrary set of values. The simulated data have 10 sellers and 100 bidders. Each seller has 2-4 items for sale and makes

²⁷Note that a seller makes decision in Stage 1 before bidder's decisions. The seller takes the bidder costs distribution into account when he calculates the expected return. Thus L^{seller} also appears in the conditional likelihood of C_{ijtn}^b

decisions about the optimal number of auctions for each item. The sellers' optimization results in a total of 213 auctions in the market. Each bidder then makes decisions about bidding activities. The true values and estimates of parameters are presented in Table 13 and indicate that the model is capable of recovering parameters with reasonable accuracy.

Table 13: Monte Carlo Simulation Results

	True Value	Median	95% Intervals
Valuation Distribution			
μ			
Constant	-4	-3.89*	(-4.18, -3.51)
Book Value	1	0.99*	(0.83, 1.15)
(Seller Feedback)	0.5	0.49*	(0.06, 0.89)
Minimum Bid	0.5	0.51*	(0.09, 0.91)
Secret Reserve Dummy	0.5	0.49*	(0.25, 0.72)
Store Dummy	0.5	0.50*	(0.29, 0.75)
Gallery Picture	0.5	0.50*	(0.24, 0.72)
Subtitle	0.5	0.51*	(0.32, 0.67)
σ			
Constant	2	2.09*	(1.98, 2.20)
Bidder Disutility			
Number of Auctions Attended	1	0.99*	(0.96, 1.02)
Lapse Since Last Winning	-0.4	-0.39*	(-0.37, -0.41)
Mean of the Individual Constant	2	2.08*	(1.36, 2.83)
Seller Acquisition Cost			
Book Value	1	0.99*	(0.94, 1.05)
Number of Listings	1.5	1.51*	(1.44, 1.55)
Cost Error Variance	1	1.10*	(0.91, 1.22)
Mean of the Individual Constant	0.50	0.56*	(0.36, 0.88)

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